8.421 Homework Assignment #10

Spring 2008, Prof. Wolfgang Ketterle, Prof. Vladan Vuletic

Due Monday, May 12, 2008

For questions or assistance with this assignment contact:
Christian Sanner, room 26-259, tel. 617-253-5926, sanner@mit.edu
office hour: Friday 5/9, 5-6 pm

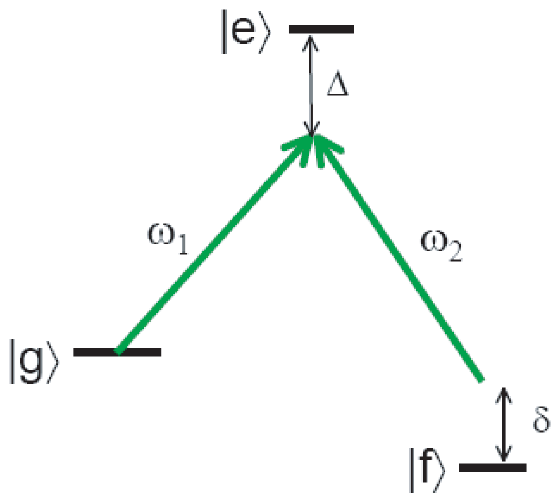
1. **Saturation spectroscopy**
This problem guides you through the concepts of saturation spectroscopy. This is one of the techniques to perform Doppler-free spectroscopy, i.e. to extract a narrow line (with the natural linewidth) in a gas with a broad velocity distribution. It nicely illustrates the combination of homogenous and inhomogeneous line broadening. Saturation spectroscopy is frequently used to lock lasers to atomic lines. You should not get into nasty integrals for this problem. The drawn lineshapes should clearly show the basic features, but don't have to be exact.

a) **Homogeneous broadening**
Consider a dilute gas of density n composed of atoms with resonant frequency ω_0 and linewidth Γ. The gas is exposed to monochromatic light of frequency $\omega_L = \omega_0 + \delta$ and intensity $I = sI_{sat}$, where I_{sat} is the saturation intensity. Let us ignore the effects of the motion of the atoms, i.e. consider temperature $T = 0$. What are the densities of atoms in the ground state n_1 and the excited state n_2, including the effect of saturation? What is the cross-section for absorption? The gas is in a box of length L along the direction of the incoming light. What fraction of the light is absorbed? (This is a small fraction, so don't worry about the effect of light attenuation on the saturation of the sample).

b) **The Bennet hole**
Now, let's endow these atoms with a mass m and a temperature T. Let the incoming light have a wavevector k_L along the z-axis. What is the population density distribution in the ground state $n_1(v_z)$ as a function of v_z, the component of velocity in the z-direction? You should find that the light "burns a hole" (known as the Bennet hole) in the distribution of absorbers. What is the position of the hole? How do the width and depth (relative to the population for $s = 0$) vary with saturation parameter s?

c) **Inhomogeneous broadening**
Consider that we sweep the frequency of the incident laser ω_L and measure the (small) absorption of the beam. Determine the fraction of the light absorbed as a function of s and and compare with its value at $s = 0$ (you don't need to solve the integral). For high temperatures ($k_L\overline{v}_z \gg \Gamma$, what is the width of the absorption line? Does saturation affect the width?

d) **Saturation spectroscopy**
To actually get some benefit from saturating the gas, we introduce a second laser beam.
i) We can add a weak probe beam at frequency ω_p with wavevector k_p. What is the absorption of this beam, including the effects of the saturating beam (k_L, ω_L)? Again, just write the integral, and take the length of the box along k_p to be L. Draw the absorption line shape, identifying the position and width of its features.
ii) We can also just retroreflect the original beam. Draw the population distribution $n_1(v_z)$ for $\omega_L \neq \omega_0$ and $\omega_L = \omega_0$. Identifying the depth of the Bennet hole(s), draw the lineshape of absorption of the retroreflected beam (i.e. we scan ω_L). What is the width of the central feature (at $\delta = 0$)?



2. **Dark state in electromagnetically induced transparency and STIRAP**

A three-level atom with states $|g\rangle, |e\rangle, |f\rangle$ driven by two laser fields with Rabi frequencies ω_1 and ω_2 on the transitions $|g\rangle \leftrightarrow |e\rangle$ and $|f\rangle \leftrightarrow |e\rangle$, respectively, is described in the rotating-wave approximation in the interaction picture by the Hamiltonian

$$H = \hbar \begin{pmatrix} 0 & \frac{1}{2}\omega_1 & 0 \\ \frac{1}{2}\omega_1 & \Delta & \frac{1}{2}\omega_2 \\ 0 & \frac{1}{2}\omega_2 & -\delta \end{pmatrix}. \quad (1)$$

a) Show that on two-photon resonance ($\delta = 0$) there is a dark state that has no light shift and no admixture of the excited state $|e\rangle$, independent of the one-photon detuning Δ. Write down the dark state in terms of the components $|g\rangle, |f\rangle$ and the Rabi frequencies ω_1, ω_2, and interpret the result.
b) Now assume one-and two-photon resonance, $\Delta = \delta = 0$. Show that the dark state can be evolved adiabatically from $|g\rangle$ to $|f\rangle$ by slowly changing the Rabi frequencies ω_1, ω_2. This process is called STIRAP (stimulated rapid adiabatic passage), and allows you to transfer atoms between two ground states without involving loss from decay of the excited state $|e\rangle$ to other levels. What is the condition for this process to be adiabatic?