8.421 Homework Assignment #6

Spring 2008, Prof. Wolfgang Ketterle, Vladan Vuletic

Due Friday, April 11, 2008

For questions or assistance with this assignment contact:
Ye-ryoung Lee, room 26-261, tel. x3-4178, yeryoung@mit.edu.
Office Hour: Thursday April 10 2-4pm, Rm 26-261

1. **Atom-Cavity Oscillations**

In lecture the interaction of a two-level atom with a single empty mode of the radiation field was discussed for the case in which the field is in resonance with the atom: $\hbar\omega = \hbar\omega_0 = E_a - E_b$. This would occur if the atom were in an ideal cavity tuned to ω_0. The problem is to find out what would happen if the cavity were tuned to some other frequency $\omega' \neq \omega_0$.

a) Does this system have stationary states? Explain.

b) The atom is placed in the cavity in its upper state, $\mid a >$. Find the probability P_a that at a later time t it is in state $\mid a >$.

c) Explain whether or not the overall energy of the system is conserved in part b).

2. **Saturation Intensity**

We refer to the saturation intensity of a laser for an optical transition as the intensity (power/area) at which a monochromatic beam excites the transition at a rate equal to one half of its natural line width (i.e., half of the maximum rate: this definition is not rigorous.) Find the saturation intensity for the principal transition in sodium, 590 nm. Treat the atom as a two-level system, neglecting fine and hyperfine structure. Take the natural lifetime of the 3P state to be 16 ns.

3. **Atom-Cavity Oscillations for a Rydberg Atom**

A Rydberg atom is an atom with an electron in a high-n state. The frequency for the transition n $\rightarrow$ n-1 is n^{-3}, and the electric dipole matrix element for the transition is n^2. (The units are atomic, and the relations approximate.)

a) Find the frequency, in Hz, of the transition n=50$\rightarrow$n=49, and the dimensions of the smallest cubical cavity which can be tuned to that resonance. (Frequency in atomic units is always in radians/time.)

b) Find the vacuum Rabi oscillation frequency for this system. Express your answer in Hz.