

## 8.421 Homework Assignment #1

Spring 2008, Prof. Wolfgang Ketterle, Prof. Vladan Vuletic

Due Monday, February 25, 2008

For questions or assistance with this assignment contact:

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office hour: Thursday, 02/21, 5-6 pm

1. [4 points] When driven far from resonance the power dissipated in a mechanical (classical) damped oscillator increases linearly with the damping  $\gamma$ , but on resonance varies as  $\gamma^{-1}$ . Why does reducing the damping increase the power dissipated on resonance?

2. [8 points] In this problem we want to study the time evolution of a system with a Hamiltonian

$$H = -\vec{\mu} \cdot \vec{B} = \frac{\hbar}{2} \begin{pmatrix} \omega_0 & \omega_R e^{-i\omega t} \\ \omega_R e^{i\omega t} & -\omega_0 \end{pmatrix}. \quad (1)$$

This Hamiltonian corresponds to a magnetic moment  $\vec{\mu}$  in a combination of static and rotating fields  $\vec{B}(t) = -(B_1 \cos \omega t, B_1 \sin \omega t, B_0)$ . Here  $\omega_0 = \gamma B_0$  and  $\omega_R = \gamma B_1$  are the Larmor and Rabi frequency associated with the static field  $B_0$  and the rotating field of magnitude  $B_1$ , respectively, and  $\gamma$  is the gyromagnetic ratio. The basis is  $\begin{pmatrix} 1 \\ 0 \end{pmatrix} = |\downarrow\rangle \equiv |e\rangle$ ,  $\begin{pmatrix} 0 \\ 1 \end{pmatrix} = |\uparrow\rangle \equiv |g\rangle$ , where  $|\uparrow\rangle, |\downarrow\rangle$  are the states where  $\vec{\mu}$  is oriented along the  $\mp z$  axis. The time evolution of any state  $|\psi(t)\rangle = a_g(t)|g\rangle e^{i\omega_0 t/2} + a_e(t)|e\rangle e^{-i\omega_0 t/2}$  is determined by the two coefficients  $a_g(t), a_e(t)$ .

a.) Find the equations of motion for the system, i.e. derive explicit expressions for  $\dot{a}_g(t)$  and  $\dot{a}_e(t)$ .

b.) Solve the equations of motion and find  $a_g(t)$  and  $a_e(t)$  in terms of  $a_g(0)$  and  $a_e(0)$ .

c.) Given the initial conditions  $a_g(0) = 1$  and  $a_e(0) = 0$  show that the probability to find the system in the state  $|e\rangle$  agrees with the classical result.

3. [8 points] Now we want to analyze the Hamiltonian of problem 2 in the density matrix formalism. Parameterize  $H$  as  $H = \frac{\hbar}{2}[V_1 \hat{\sigma}_x + V_2 \hat{\sigma}_y + \omega_0 \hat{\sigma}_z]$ , and the density matrix as  $\rho = \frac{1}{2}[r_0 \hat{1} + r_1 \hat{\sigma}_x + r_2 \hat{\sigma}_y + r_3 \hat{\sigma}_z]$ , where  $\hat{1}$  is the unity matrix, and

$$\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \hat{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \hat{\sigma}_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (2)$$

are the Pauli spin matrices.

Employing the von Neumann equation  $i\hbar \dot{\rho} = [H, \rho]$  show that  $\vec{r} = r_1 \hat{x} + r_2 \hat{y} + r_3 \hat{z}$  obeys the relation  $\frac{d\vec{r}}{dt} = \vec{\omega} \times \vec{r}$  with  $\vec{\omega} = V_1 \hat{x} + V_2 \hat{y} + \omega_0 \hat{z}$ . Can you interpret this result?