8.421 Fall 2008 Assignment #8
Professors W. Ketterle and V. Vuletić
Due Monday, April 28, 2008

For questions or assistance with this assignment, contact:
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Office Hours (in room 26-217): Friday April 25, 2008, 9am-11am, or by appointment

1. **Line Shape Due to a Fluctuating Field** (10 pts.)

   This problem is somewhat artificial, but it provides an exercise in evaluating a correlation function and bears on related non-resonance phenomena such as relaxation in a fluctuating field.

   A two level system, with states $|a\rangle$ and $|b\rangle$ that have energy separation $\hbar\omega_0$, is subjected to an oscillating perturbation, for instance the field of laser light, with a matrix element of the form

   $$\langle b|V(t)|a\rangle = \frac{x}{2}e^{-i\omega t}$$

   The two levels decay with a decay rate $\gamma/2$. Furthermore, the field amplitude x is not constant but flips between two values, $+A$ and $-A$. The flipping is random, occurring with a mean rate Γ. The problem is to find the resonance line shape. This requires finding the correlation function $G_{ba}(\tau)$.

   As a guide, consider the probabilities $p_+(\tau)$ and $p_-(\tau)$ which represent the probability that if x is $+A$ at $t = 0$ it will be $+A$ or $-A$ at $t = \tau$, respectively. The correlation function can be expressed in terms of $p_+(\tau)$ and $p_-(\tau)$. From the coupled rate equations for $p_+(\tau)$ and $p_-(\tau)$, $G_{ab}(\tau)$ can be found, and from this the line shape.

2. **Intensity Distribution Due to Spontaneous Emission** (10 pts.)

   An atom of total angular momentum J has a spontaneous radiation rate A. It radiates to a lower level with angular momentum $J' = J - 1$. The problem is to find the rates for the various allowed transitions, i.e. the fraction of the radiation that goes into each of the possible transitions $(J, m) \longrightarrow (J', m')$. The rates can be found by applying the following considerations:

   - The sum of the rates out of each state (J, m) must equal A.
   - The sum of the rates into each state (J', m') must equal $A \times \frac{2J+1}{2J'+1}$.
   - An unpolarized mixture of radiators in level J must emit equal intensities of light with each of the three polarization components.
   - The rate for a transition $(J, m) \longrightarrow (J', m')$ must be the same as for $(J, -m) \longrightarrow (J', -m')$.

For $J = 2,\ J' = 1$, designate the transitions by letters as follows:

$$
\begin{aligned}
a&: \quad m = 2 \longrightarrow m' = 1 \\
b&: \quad m = 1 \longrightarrow m' = 1 \\
c&: \quad m = 0 \longrightarrow m' = 1 \\
d&: \quad m = 1 \longrightarrow m' = 0 \\
e&: \quad m = 0 \longrightarrow m' = 0
\end{aligned}
$$

(a) Find the rates for a though e, and present your results on a figure.

(b) Find the rates for a though e, using the Wigner-Eckart theorem. (Clebsch-Gordan coefficients can either be worked out from first principles, or taken from a table in one of the quantum mechanics or spectroscopy texts.)

Note that the transition rates calculated here are important in experiments involving laser excitation. Because emission and absorption rates are proportional, the distribution of emission rates yields the relative strengths of the transitions, i.e. their relative rates of excitation.