

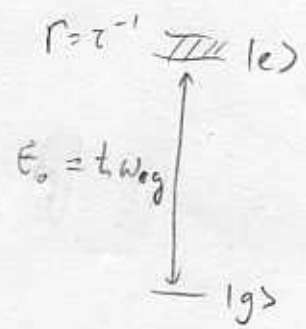
Atom-light interaction: outline

- Broadband and narrowband excitation
 - Broadband excitation: Einstein rate equations
 - Strong narrowband excitation: Rabi flopping
- qm derivation of Einstein B, A coefficients
- Quantitation of em fields
- qm description of spontaneous emission: Weisskopf-Wigner theory
- Line strength; selection rules
- Beyond the dipole approximation: higher-order processes

Atom-light interaction

An atom in an excited state left in a vacuum of the electromagnetic field (or a very large volume with walls kept at sufficiently low temperature) decay irreversibly to the ground state in a time τ that is long compared to the oscillation period $\frac{2\pi}{\omega_0}$.

For electronic excited states, corresponding to optical transitions, the low-temperature condition on the walls is easily fulfilled, $k_B T \ll \hbar \omega_0$,



since room temperature corresponds to $k_B T_{\text{room}} = 25 \text{ meV}$, while $\hbar \omega_0 \sim$ a few eV for optical transitions.

The inverse lifetime $\tau^{-1} = \Gamma$, or natural linewidth of the transition, sets a frequency scale. The interaction between light and atoms is particularly simple in two limiting cases

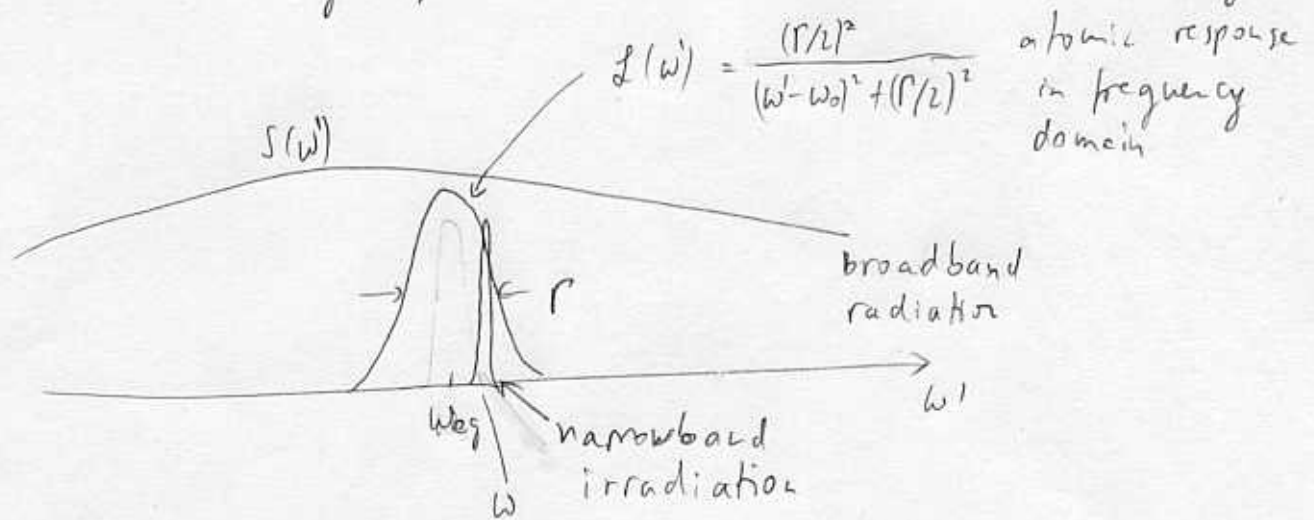
$$\Delta \omega \ll \Gamma$$

narrowband excitation with limit $\Delta \omega \rightarrow 0$
(monochromatic irradiation)

$$\text{and } \Delta \omega \gg \Gamma$$

broadband irradiation,

where $\Delta\omega$ is the frequency scale over which the spectral density of the radiation varies appreciably



For broadband excitation, the light is characterized by the spectral density of radiation at the atomic frequency

$$S(\omega) = \frac{[\text{energy}]}{[\text{volume}] \times [\text{frequency interval}]}$$

or intensity per unit frequency interval

$$I(\omega) = c S(\omega) = \frac{[\text{energy}]}{[\text{area}] \times [\text{time}] \times [\text{frequency interval}]}$$

For monochromatic radiation the light is specified by frequency ω and electric field amplitude E (or intensity I).

Quantum mechanically, the narrowband case (excitation by a monochromatic light field) is simpler.

The simplest case is the idealized situation where the atom interacts with a single-mode only - in emission as well as in absorption. We then obtain an oscillatory exchange of energy between atom and light field - almost identical to the problem of the spin in a combination of static and rotating magnetic fields. A slightly more complicated situation arises when the atom is driven by a single-mode field, but can couple in emission to a continuum of modes.

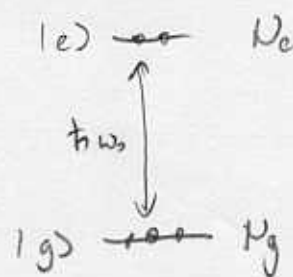
In this case, that can be treated using Fermi's golden rule, the coherent, oscillatory, and reversible energy exchange process gives way to an incoherent, irreversible process described by rate equations (Weisskopf - Wigner theory of spontaneous emission).

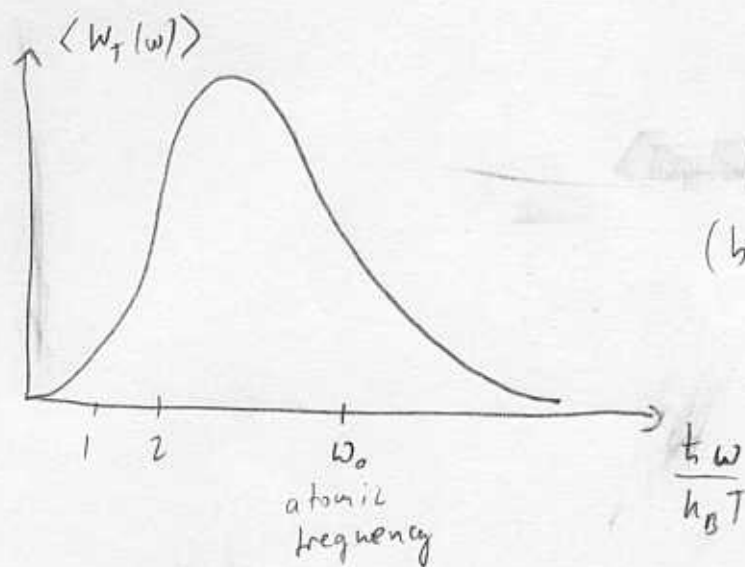
Finally, the case of broadband excitation is obtained by averaging over many exciting modes.

The case of broadband excitation can alternatively be understood using a semiclassical treatment based on postulates that he introduced in order to explain how an atom could come into thermal equilibrium with a thermal radiation field. The corresponding concepts of (stimulated) absorption, stimulated emission and spontaneous emission remain important even in the context of QM in order to understand processes governed by rate equations. The laser is an example of such a process.

Einstein's A and B coefficients

Consider a box containing thermal radiation at temperature T , and N atoms of which N_g and N_e are in the ground and excited state, respectively.





Planck spectrum
(blackbody spectrum)

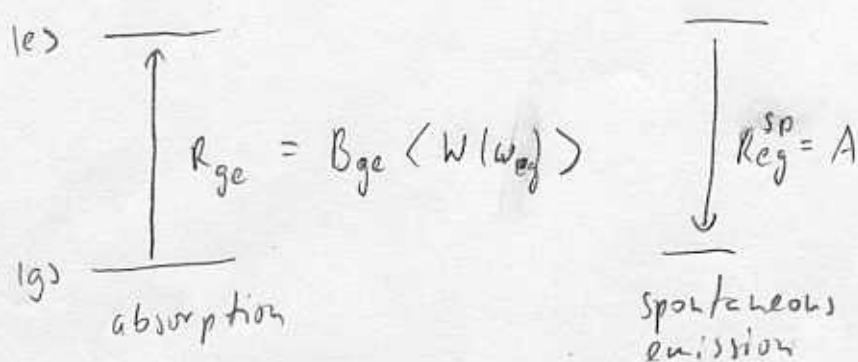
$$\underbrace{\langle W_T(\omega) \rangle}_{\substack{\text{mean energy} \\ \text{per unit volume} \\ \text{in frequency} \\ \text{interval } [\omega, \omega+d\omega] \\ [J/m^3]}} d\omega = \underbrace{\langle n(\omega) \rangle}_{\substack{\text{photon} \\ \text{number} \\ \text{per mode} \\ \text{at } \omega \\ \text{(per unit} \\ \text{volume)}}} \hbar\omega \underbrace{p(\omega)}_{\substack{\text{density of} \\ \text{modes} \\ \text{(modes per unit} \\ \text{frequency interval)} \\ \text{at } \omega \text{ (per unit volume)}}} d\omega = \frac{\hbar\omega^3}{\pi^2 c^3} \frac{d\omega}{\exp(\frac{\hbar\omega}{k_B T}) - 1}$$

with $\langle n(\omega) \rangle = \frac{1}{\exp(\frac{\hbar\omega}{k_B T}) - 1}$

This law is easily obtained by requiring that the probabilities for mode populations follow Boltzmann's law, i.e. that the probability ratio for finding m and n photons in the mode is given by

$$\frac{P_n}{P_m} = e^{-\frac{(E_n - E_m)}{k_B T}}$$

It seems natural to assume that the rate at which an atom makes a transition $|g\rangle \rightarrow |e\rangle$ (absorbs a photon) is proportional to the photon or energy density at the frequency $\omega = \omega_0$.
 Let us call the proportionality constant B_{ge}



The atoms must also come down. Let us assume that they come down at constant rate A . Then

we would obtain an equilibrium governed by

$$N_g B_{ge} = N_e A \quad \text{or} \quad \frac{N_e}{N_g} = \frac{B_{ge} \langle W(\omega_0) \rangle}{A} \propto \frac{\omega_{eg}^3}{\exp(\frac{\hbar\omega_{eg}}{k_B T}) - 1}$$

This is clearly not the Boltzmann distribution

in particular, for $\hbar\omega \gg k_B T$, we have $\frac{N_e}{N_g} \propto \omega_{eg}^3 e^{-\frac{\hbar\omega_{eg}}{k_B T}}$,
 i.e. the excited state population is too large. We need an additional process $|e\rangle \rightarrow |g\rangle$.

Einstein postulated a stimulated emission rate

$$R_{eg} = B_{eg} \langle W(\omega_{eg}) \rangle$$

in addition to the spontaneous rate $A = R_{eg}^{sp}$.

This seems very reasonable: do the photons really care when interacting with the atoms which of the states $|g\rangle, |e\rangle$ is lower or higher in energy? Then the rate equation for the populations has an equilibrium governed by

$$N_g B_{ge} \langle W(\omega_{eg}) \rangle = N_e B_{eg} \langle W(\omega_{eg}) \rangle + N_e A \quad \text{or}$$

$$\begin{aligned} \frac{N_g}{N_e} &= \frac{B_{eg}}{B_{ge}} + \frac{A}{B_{ge} \langle W(\omega_{eg}) \rangle} = \frac{B_{eg}}{B_{ge}} + \frac{A}{B_{ge}} \frac{\pi^2 c^3}{\hbar \omega_{eg}^3} (\exp(\frac{\hbar \omega_{eg}}{kT}) - 1) \\ &= \frac{A}{B_{ge}} \frac{\pi^2 c^3}{\hbar \omega_{eg}^3} \exp(\frac{\hbar \omega_{eg}}{kT}) + \frac{B_{eg} - A \frac{\pi^2 c^3}{\hbar \omega_{eg}^3}}{B_{ge}} \end{aligned}$$

If we want this to be just the Boltzmann factor that governs atomic populations in thermal equilibrium we must require

$$\boxed{B_{eg} = B_{ge} = A \frac{\pi^2 c^3}{\hbar \omega_{eg}^3}} ,$$

Thus the stimulated absorption and emission coefficients