Lecture XV

Scattering cross section

We have derived the formula for the photon scattering rate in the case of weak narrowband excitation,

$$R_{sc} = \frac{\omega_R^2}{\Gamma} \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + \Delta^2}$$

in terms of the Rabi frequency $\omega_R = \sqrt{n+1}\,\omega_1$

and the single-photon Rabi frequency $\omega_1 = -\frac{2}{\hbar} \vec{d}\,\hat{e} \sqrt{\frac{\hbar \omega_{eg}}{2\varepsilon_0 V}}$

To express this in terms of classical-field properties, we note that (using $(\hat{d}\cdot\hat{e})^2 \to d_x^2 = \frac{1}{3} \vec{d}^2$,

$$\omega_R^2 = (\vec{d}\cdot\hat{e})^2 \frac{2\omega_{eg}(n+1)}{\varepsilon_0 \hbar V} = \underbrace{\frac{\vec{d}^2 \omega_{eg}^3}{3\pi\varepsilon_0\hbar c^3}}_{3\Gamma} \frac{2\pi c^3}{V\omega_{eg}^2}(n+1) = \Gamma^2 \frac{6\pi c^3}{V\omega_{eg}^2 \Gamma}(n+1)$$

where $A = \Gamma = \frac{g_g}{g_e} \frac{\vec{d}^2 \omega_{eg}^3}{3\pi\varepsilon_0\hbar c^3} = \frac{\vec{d}^2\omega_{eg}^3}{9\pi\varepsilon_0\hbar c^3}$ for an $S-P$ transition

For large n, $n+1 \approx n$, $V =$ area $\cdot$ length, time $= \frac{\text{length}}{c}$

$$I = \frac{\text{energy}}{\text{area}\cdot\text{time}} = \frac{\hbar\omega n}{V/c} \approx \frac{\hbar c\,\omega(n+1)}{V} = \frac{\hbar c\,\omega_{eg}(n+1)}{V}$$

$$\frac{\omega_R^2}{\Gamma^2} = I \frac{6\pi c^2}{\hbar \omega_{eg}^3 \Gamma}$$

The last term has units of $(\text{intensity})^{-1}$, we define

$$I_s = \frac{\hbar \omega_{eg}^3 \Gamma}{12\pi c^2} \qquad \text{saturation intensity}$$

This allows us to express the Rabi frequency as

$$\omega_R^2 = \frac{I}{2 I_s} \Gamma^2$$

and the scattering rate becomes

$$R_{sc} = \frac{\omega_R^2}{\Gamma} \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + \Delta^2} = \frac{I}{I_s} \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + \Delta^2} \frac{\Gamma}{2}$$

We have derived the formula for $\omega_R^2 \ll \Gamma^2$, i.e. for $I \ll I_s$. It remains valid for

$$p := \frac{I}{I_s} \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + \Delta^2} \ll 1$$

p is called the <u>saturation parameter</u>

The scattering cross section is the inverse ratio intensity and scattered power. On resonance we have

$$\sigma_0 = \frac{\hbar \omega_{eg} R_{sc}}{I} = \frac{\hbar \omega_{eg} \Gamma}{2 I_s} = \frac{6\pi c^2}{\omega_{eg}^2} = \frac{6\pi}{k^2} = 6\pi ƛ^2 = \frac{3}{2\pi} \lambda^2$$

$$\sigma_0 = 6\pi ƛ^2 = \frac{3}{2\pi} \lambda^2 \qquad \text{resonant cross section}$$

$\left(ƛ = \frac{\lambda}{2\pi}\right)$. Note that the resonant cross section is a) independent of linewidth and b) much larger than the atomic size.

The off-resonant scattering cross section is

$$\sigma(\Delta) = \sigma_0 \frac{(\Gamma/2)^2}{(\Gamma/2)^2 + \Delta^2}$$

The saturation intensity can be rewritten as

$$I_s = \frac{\hbar \omega_{eg} \Gamma/2}{\frac{3}{2\pi} \lambda^2} = \frac{\hbar \omega_{eg} \Gamma/2}{\sigma_0}$$

ali 55