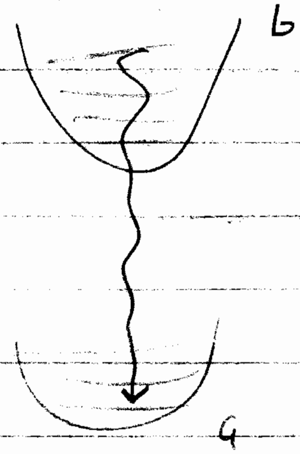
Lineshape of confined particles

Trapped particles → ultimate limit in spectroscopic precision

cooling reduces second order Doppler shifts

first order Doppler shift is suppressed by the trapping pot.

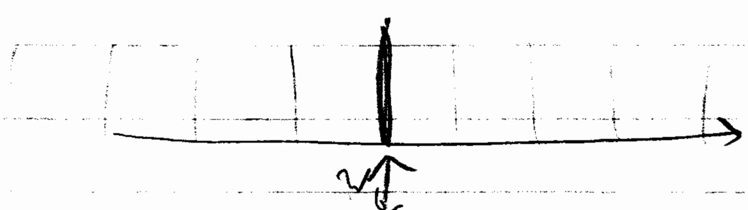
1) Spectrum of oscillating emitter



$\hbar\omega = \hbar\omega_{ba} + E_i^{trap} - E_f^{trap}$ general

$= \hbar(\omega_{ba} + n\omega_t)$ harmonic motion

recoil included, since E_i^{trap}, E_f^{trap} include kinetic energy.



Sideband spectrum

no broadening, but discrete sidebands!

Rate $\propto$ |matrix element|^2 $\propto \left| \langle \psi_f^{trap} | e^{-ikr} | \psi_i^{trap} \rangle \right|^2$

$1 + ikr$

Intensity $k^2 \langle r^2 \rangle = \langle r^2 \rangle / \bar{\lambda}^2$ $\Delta n = \pm 1$

classical:

oscillating emitter or absorber

$e^{-ikx} e^{-i\omega_0 t}$ plane wave vs Lab freq

atom res phase $\phi(t) = -k\,x(t) - \omega_0 t = -k\, x_0 \sin\omega_t t - \omega_0 t$

instantaneous Freq: $\omega(t) = -\dot{\phi}(t) = k\,v(t) + \omega_0$

normal Doppler shift; one could now determine the time δt when the frequency is $\omega(t)$ and calculate the spectrum.

However, we should perform the Fourier transform of the total wave train.

$$a(t) = \cos\phi(t) = \cos(\omega_0 t + \beta \sin\omega_t t)$$

amplitude of wave $\qquad \beta = k x_0 = x_0/\lambda\!\!\!^{-}$ modulation index

expansion into Bessel Function

Use identities

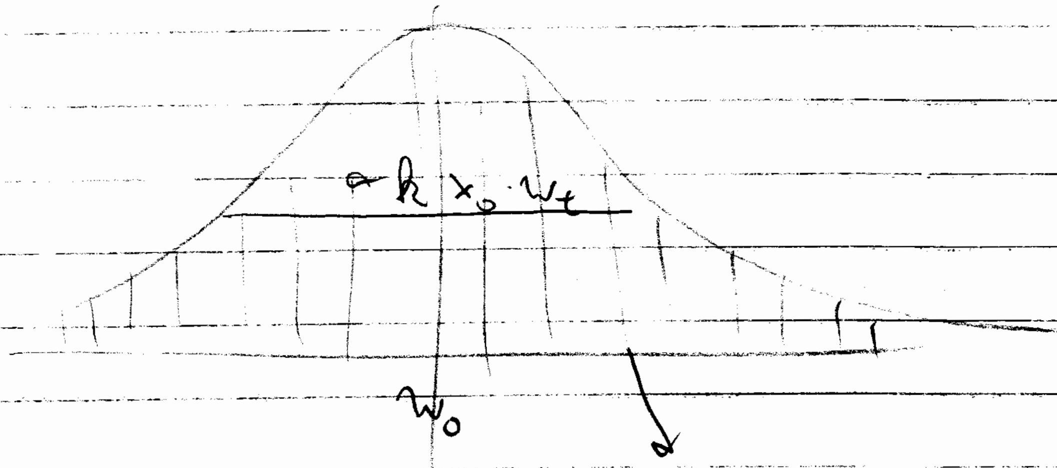
$$\cos(z\sin\theta) = J_0(z) + 2\sum_{l=1}^{\infty} J_{2l}(z)\cos(2l\theta)$$

$$\sin(z\sin\theta) = 2\sum_{l=0}^{\infty} J_{2l+1}(z)\sin[(2l+1)\theta]$$

$$\Rightarrow \quad a(t) = \sum_{n=-\infty}^{\infty} (-1)^n J_n(\beta)\cos[(\omega_0 + n\omega_t)t]$$

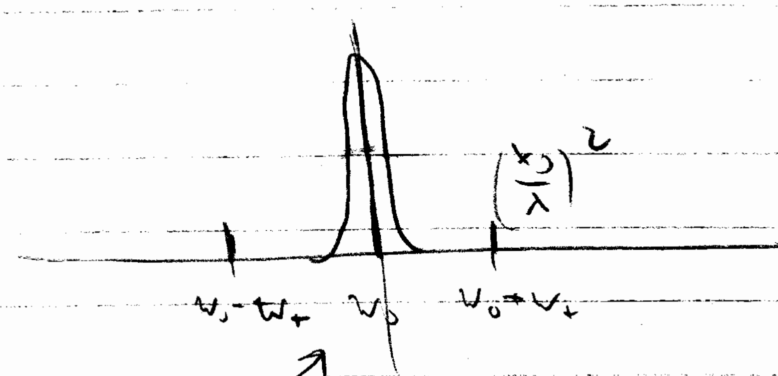
$$I(\omega) \propto \sum_{n=-\infty}^{\infty} J_n^2(\beta)\,\delta(\omega - \omega_0 - n\omega_t)$$

How does the spectrum look like



if $\omega_t < \frac{\Gamma}{\hbar}$: side bands not resolved

$x_0 \ll \lambda$ (Lamb–Dicke regime



no motional broadening, sharp lines
recoilless emission i.e. no change of motion of the atom

trap takes over the momentum
analogy to Mössbauer effect
Intensity in central peak $\propto$ (matrix element)^2
analogous to the Debye-Waller factor

Correlation function formalism:

$$W_{ba} = G(\omega) = (\ ) |V(\omega)|^2$$

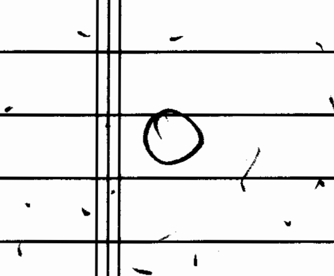
$$V(t) = e^{-ikx_0 \sin \omega_t t} \, e^{-i\omega t}$$

$\Rightarrow$ Bessel functions

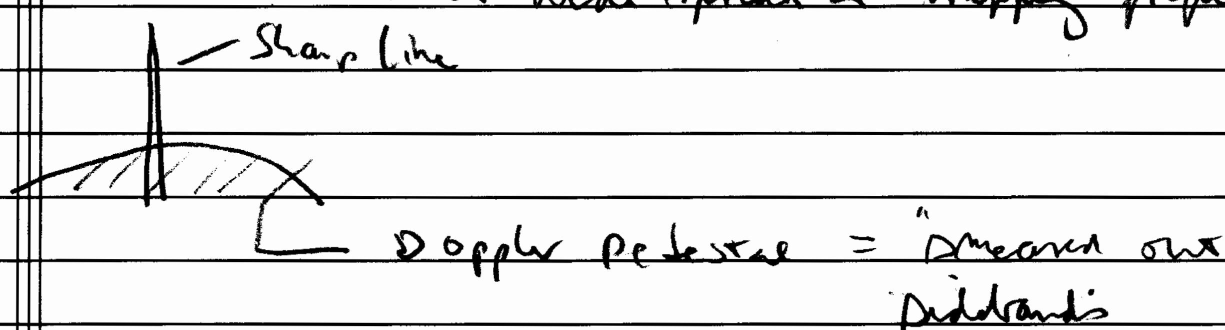
Interpretation:
What matters is the accumulated phase.
If atoms in the ensemble accumulate phases differing by 2π in a time Δt then this is the correlation time τ_c determining the line broadening.

If atoms are in rapid motion, but they turn around before they spread out by a wavelength $\Rightarrow$ $+kv$ and $-kv$ cancel, what matters is the "integrated" Doppler shift, which is zero for confined particles.

Dicke narrowing



Atom in buffer gas
In some cases (noble gases) collisions don't change internal coherence (i.e. phase between $|g\rangle$ and $|e\rangle$)
$\Rightarrow$ buffer gas acts as a "trap", with wide spread in trapping frequencies



Let's estimate the width of the sharp peak.

atom diffuse $z_{rms}^2 = 2Dt$

diffuses over by $\bar{\lambda}$ after $t = \frac{\bar{\lambda}^2}{2D} = \frac{1}{2k^2 D}$

$\Rightarrow$ FWHM $\propto k^2 D$

is NOT correct at very short time
$\Rightarrow$ Doppler pedestal is missing

Let's use the correlation function

$$G_{ba} = \frac{|x|^2}{4} \overline{e^{-iks}} \, e^{i\omega t} \qquad P(s,t) = \frac{1}{\sqrt{4\pi Dt}} e^{-\frac{s^2}{4Dt}}$$

$$= \frac{|x|^2}{4} \frac{1}{\sqrt{4\pi Dt}} \int_{-\infty}^{\infty} e^{-iks} \, e^{-s^2/4D\tau} ds \; e^{i\omega\tau}$$

$$= \frac{|x|^2}{4} e^{-k^2 Dt} e^{i\omega\tau}$$

exp decaying function

$$W_{ba} = G_{ba}(\omega_0) = \frac{|x|^2}{4} 2\,\mathrm{Re}\left(\frac{1}{k^2 D + i(\omega_0 - \omega)}\right)$$

$$= \frac{|x|^2}{2} \frac{k^2 D}{(k^2 D)^2 + (\omega_0 - \omega)^2}$$

Lorentzian

with $\Delta\omega_{Dicke} = 2k^2 D$ ideal gas $D = \bar{v}\,\ell/3$

average speed $\simeq v$ → mean free path

$\Delta\omega_{Dicke} = \frac{2}{3} k \alpha \frac{\ell}{\bar{\lambda}} \sim \Delta\omega_{Doppler} \times \frac{\ell}{\bar{\lambda}} \ll \Delta\omega_{Doppler}$ if $\ell \ll \lambda$