Higher order radiation processes

$$H_{int} = -\frac{e}{mc}\vec{p}\,\vec{A} + \frac{e^2}{2mc^2}|\vec{A}|^2 + g_s\mu_B\,\vec{S}\cdot(\nabla\times\vec{A})$$

so far only considered for static B fields

atom field interaction, $\vec{p}\,\vec{A}$-term:

$$H_{ba} = \frac{e}{mc}\langle b|\,\vec{p}\cdot\vec{A}(r)\,|a\rangle$$

$$A(r) = A\,\hat{z}\,e^{ikx}$$ now: expansion

$$H_{ba} = \frac{eA}{mc}\langle b|\,p_z\left(1 + ikx + \frac{1}{2}(ikx)^2 + \dots\right)|a\rangle$$

dipole term;

if dipole radiation is forbidden, for instance if $|a\rangle$ and $|b\rangle$ have the same parity, the second term becomes important. Usually, it is α times smaller

$$kr = \frac{\hbar\omega}{\hbar c}a_0 \approx \frac{e^2}{\hbar c} = \alpha \quad \text{expansion in } \alpha$$

$$\hbar\omega = \frac{e^2}{a_0}$$

2nd term: $$p_z x = \underbrace{\frac{1}{2}(p_z x - z p_x)}_{-\frac{1}{2}\hbar L_y} + \frac{1}{2}(p_z x + z p_x)$$

μ_B

$$-\frac{i e\hbar A k}{2mc}\langle b|L_y|a\rangle = \underbrace{B\cdot\mu_B\langle b|\vec{L}|a\rangle}_{-\vec{\mu}}$$

$$B = -ikA\,\hat{y}$$

$$H_{int}(M1) = \mu_B\vec{B}\,(\langle b|\,\vec{L} + \vec{S}g_s|a\rangle$$

"magnetic dipole transition"

where we crossed here add the spin dependent term

Strength of M1 transition

2

$\mu_B = \frac{1}{2}\frac{e\hbar}{mc} = \frac{1}{2}\underbrace{\frac{e^2}{\hbar c}}_{\alpha}\underbrace{\frac{\hbar^2}{em}}_{e\cdot a_0}$

using $[r, H_0] = i\hbar\, p/m$

Second Term $\quad \frac{1}{2}(p_z x + z p_x) = \frac{m}{2i\hbar}\left([z, H_0]x + z[x, H_0]\right)$

$\cancel{zH_0x} - H_0 zx + zxH_0 - \cancel{zH_0x}$

$= \frac{m}{2i\hbar}\left(-H_0 zx + zx H_0\right)$

Contributes to H_{ba}

$\frac{ieA}{mc}\frac{km}{2\hbar i}\langle b | -H_0 zx + zx H_0 | a\rangle = -\frac{eAk}{2c}\frac{E_b - E_a}{\hbar}\langle b | zx | a\rangle$

$= \frac{ieE\omega}{2c}\langle b | zx | a\rangle$

$\uparrow$

$E = i k A$

$$H_{int}(E2) = \frac{ie\omega}{2c}\langle b | zx | a\rangle \cdot E$$

$\langle b | \vec{r} : \vec{r} | a\rangle$

tensor

$$H^{(2)}_{int} = \underbrace{H_{int}(M1)}_{\text{real}} + \underbrace{H_{int}(E2)}_{\text{imaginary}} \Rightarrow \text{no interference}$$

$$|H^{(2)}_{int}|^2 = |H_{int}(M1)|^2 + |H_{int}(E2)|^2$$

Summary

		Operator	Parity
Electric Dipole	E1	$-e\vec{r}$	$-$
Magnetic Dipole	M1	$-\mu_B(\vec{L}+g_s\vec{S})$	$+$
Electric Quadrupole	E2	$-e\,\vec{r}:\vec{r}$	$+$

M1, E_2 $\alpha^2 = 5\times10^{-5}$ weaker than an E1 transition

"forbidden" process

Selection rules

forbidden transitions
are weaker by α^n
require higher approximation
Ex: E2, M1
Multi-photon process
relativistic effects (e.g. S→T in He)
Hyperfine interactions with the nucleus

Matrix elements — introduce spherical tensor representation for operators

$\langle nJM | T_{\ell,m} | n'J'M' \rangle =$ — every operator can be written as a sum of spherical tensors

$T_{\ell,m}$ ↓ spherical tensor of rank ℓ
(transforms under rotations like the spherical harmonic $Y_{\ell,m}$)

$$= \underline{\frac{\langle nJ \| T_\ell | n'J' \rangle}{\sqrt{2J+1}}} \; \langle J'\ell, M'm | JM \rangle$$

Clebsch Gordon coeff.

reduced matrix element

triangle rule $|J'-J| \le \ell \le |J'+J|$
$M = M' + m$

E_1, M_1

$\langle a|\vec{r}|b\rangle$ $\quad$ $\langle a|\vec{L}|b\rangle$ $\quad$ vector

dipole selection rules $\quad |\Delta J| = 0, 1$

$\Delta m = 0, 1$

$$\left.\begin{array}{ll} E_1 & \text{opposite} \\ M_1 & \text{same} \end{array}\right\} \text{ parity}$$

E2: spherical tensor of rank 2

e.g. $\quad xz = (T_{2,-1} - T_{2,1})/4$

$\Rightarrow$ selection rules for quadrupole transitions
($\hat{=}$ selection rule for operator $T_{2,m}(\vec{r})$)

$|\Delta J| = 0, 1, 2 \qquad$ not $0 \leftrightarrow 0$

$\Delta m = 0, 1, 2$