Rabi frequencies, the dark state is predominantly the state with the weaker coupling. For, e.g., $\omega_1 = 0$, the dark state is trivially $|D\rangle = |g\rangle$.

$\boxed{\text{Lecture XXIII}}$

<u>Clarification on coherence and dipole moment</u>

When considering coherence in the atom (after coherent excitation with a short pulse), or in the light, we have to be careful about the basis used.
In the atom with states $|g\rangle, |e\rangle$, clearly the coherence is maximum for $c_e = c_g = \frac{1}{\sqrt{2}}$, i.e. for a $\pi/2$ pulse. Since emission maps atomic states onto photon number states as $|g\rangle \to |0\rangle$, $|e\rangle \to |1\rangle$ (considering only a single em mode, i.e. an atom strongly coupled to a cavity), a $\frac{\pi}{2}$ pulse also maximizes the coherence <u>between photon number states $|0\rangle, |1\rangle$</u>. On the other hand, for continuous excitation we know that the

Light is coherent in the Rayleigh scattering limit (i.e. when the frequency spectrum of the scattered light is a δ-function at the incident frequency, for an infinitely heavy atom), while saturation of the atom leads to emission of increasingly incoherent light (Mollow triplet). Monochromatic, coherent light is represented qm-ly by a coherent state $|\alpha\rangle$ that has a Poissonian distribution of photon numbers.

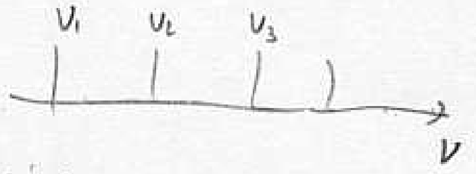
Thus in the full basis of photon number states light is coherent for the state $|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$

For $|\alpha| \ll 1$, the population of the states with $n > 1$ is negligible, and the atom prepared in a state $|g\rangle + \varepsilon|e\rangle$ with $|\varepsilon| \ll 1$ emits a coherent state of light (albeit with very small electric-field amplitude), in agreement with what is expected for small saturation.

Back to dark states in a Λ-system.

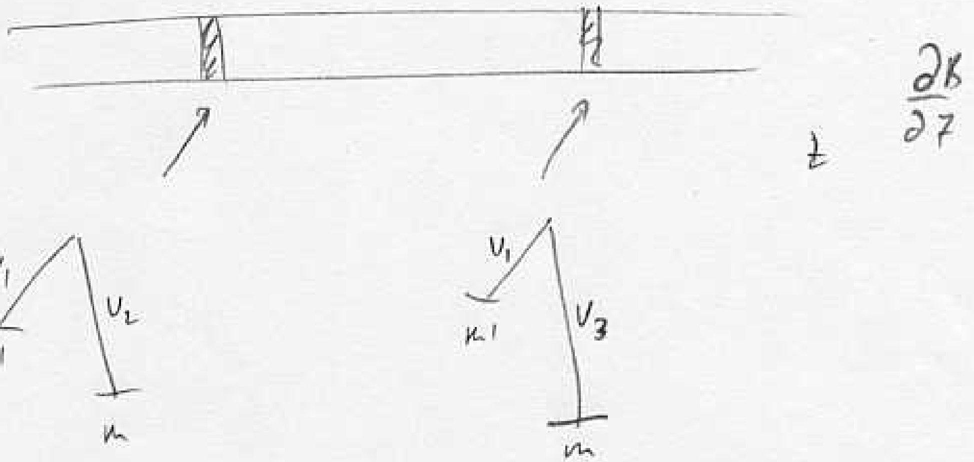
First observation of coherent population trapping CPT:

Multimode laser



with regular frequency spacing

Gas in cylindrical volume with gradient of magnetic field applied, observe fluorescence



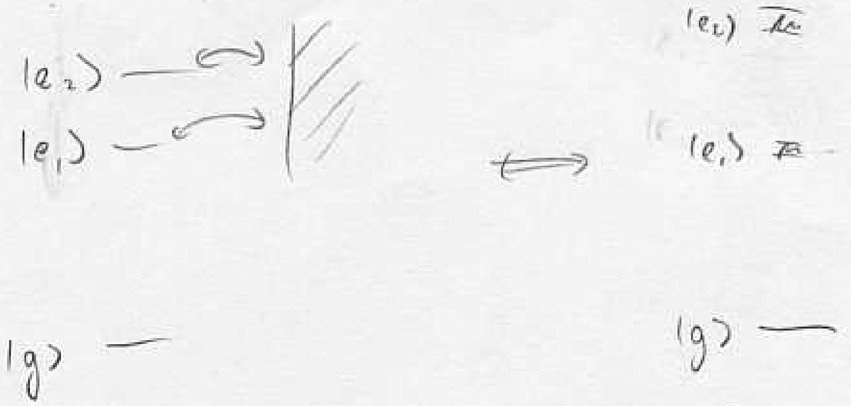
dark region where Zeeman shift between magnetic sublevels equals frequency difference between laser modes.

Absorption cancellation by interference, gain without inversion

(Steve Harris, PRL **62**, 1033 (1989))

It is commonly believed that we need $N_e > N_g$ for optical gain.

But: Consider a V system with two unstable states that decay by coupling to the <u>same</u> continuum.

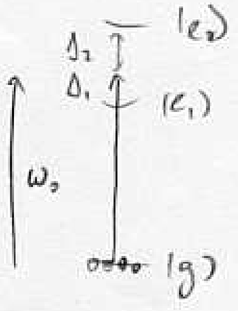


i.e. if an atom is placed in $|e_1\rangle$ or $|e_2\rangle$ and decays to the continuum, it is impossible to tell whether it came from $|e_1\rangle$ or $|e_2\rangle$.

(This is a fairly special situation, e.g. different m-levels do not qualify, since they emit photons of different polarizations, thus the continua with $|k_1\rangle$ or $|k_2\rangle$ are distinguishable.) Then the two-photon scattering process $|g\rangle \to |\text{continuum}\rangle$ can proceed via two pathways that are fundamentally indistinguishable,

$$\begin{tikzcd}
 & {|e_1\rangle} \arrow[dr] & \\
{|g\rangle} \arrow[ur] \arrow[dr] & & {|\text{continuum}\rangle} \\
 & {|e_2\rangle} \arrow[ur] &
\end{tikzcd}$$

,

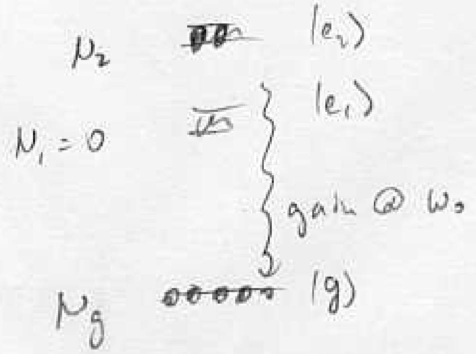


and we must add the amplitudes

The second order matrix element in perturbation theory

$$M = \sum_i \frac{\langle \text{continuum}|V|e_i\rangle \langle e_i|V|g\rangle}{\Delta_i - i\Gamma_i/2}$$

vanishes (almost exactly) for a certain frequency ω_0 that corresponds to an energy between the two levels. ω_0 depends on the two matrix elements and levels. We assume $|\Delta_1|, |\Delta_2| \gg \Gamma_1, \Gamma_2$. Then that frequency is not absorbed by atoms in $|g\rangle$, although it would be absorbed if there was only a single excited level. Now assume that with some mechanism we populate, say, $|e_2\rangle$ with a small number of atoms $N_2 < N_g$.



These atoms have maximum stimulated emission probability on resonance, $|e_2\rangle \to |g\rangle$, but there is also even larger absorption, since $N_g > N_2$. However, because of the finite linewidth Γ_2 of level $|e_2\rangle$, there is also stimulated emission gain at the "magical" (absorption-free) frequency ω_0. Since the N_g atoms do not absorb here, there is net gain at this frequency in spite of $N_2 < N_g$, which can lead to "lasing without inversion".

Note: this only works if the two excited states decay to the same continuum, such that the paths are indistinguishable.

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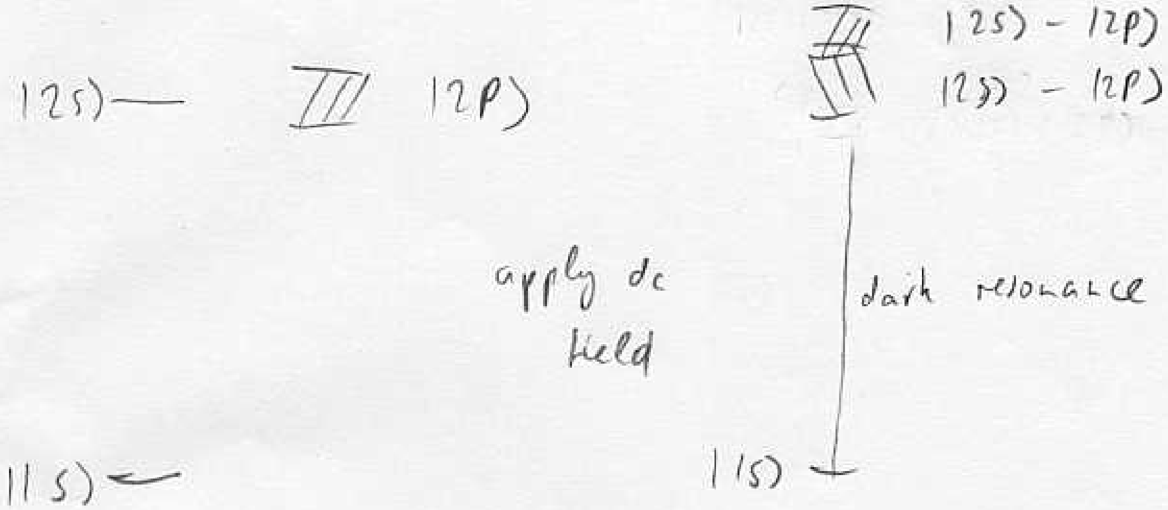
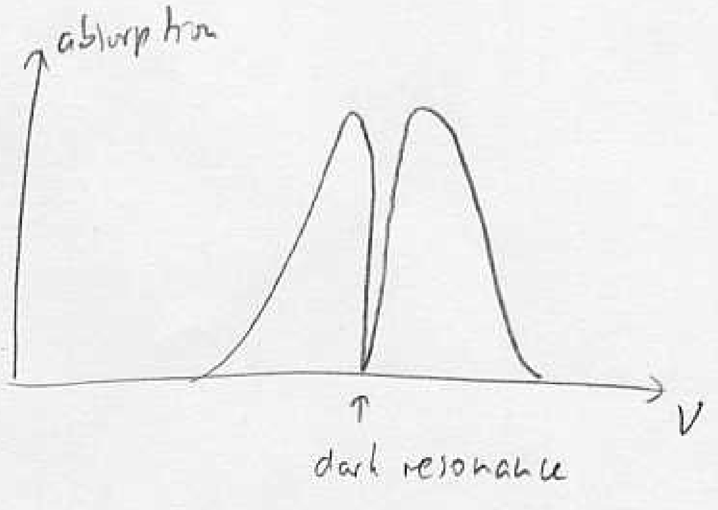
How can a system for lasing without inversion be realized?

Possibility 1: Hydrogen and dc electric field





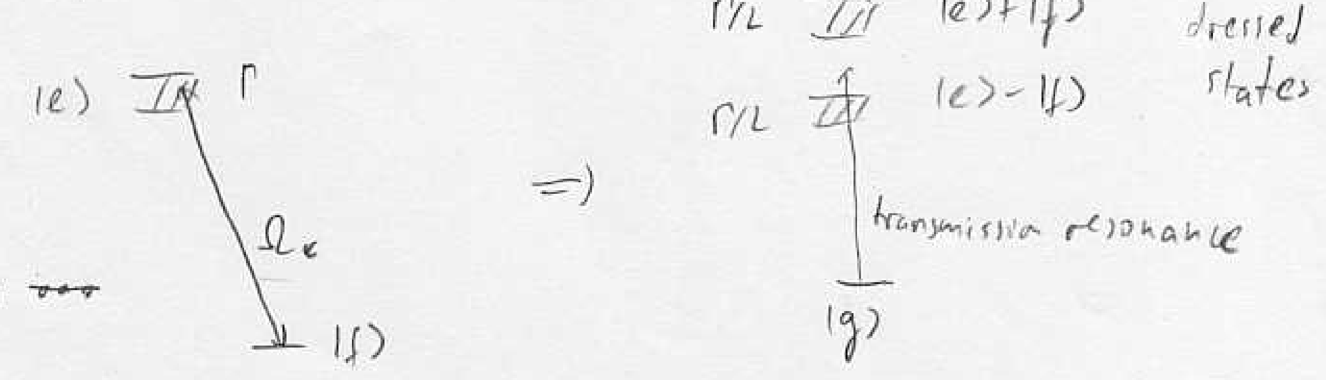
Possibility 2: use ac electric field to mix non-degenerate S state with P state.

Electromagnetically induced transparency

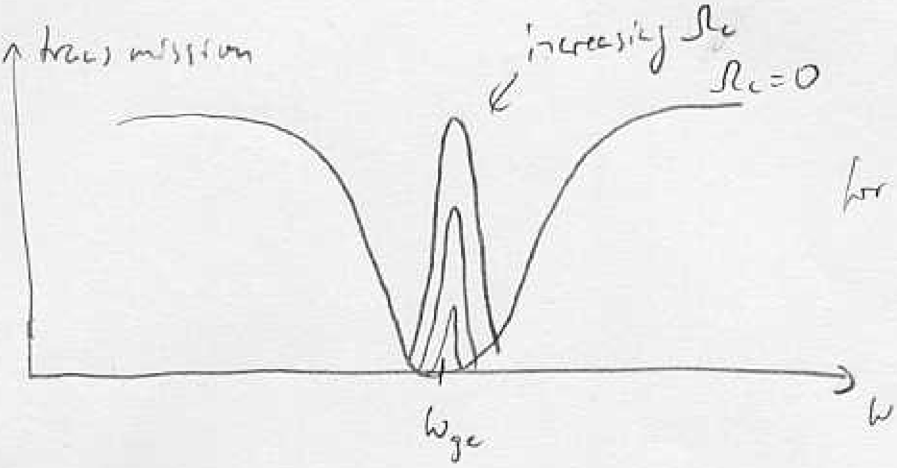
"Is it possible to send a laser beam through a brick wall?"

Radio Yerevan: "In principle yes, but you need another very powerful laser..."

Steve Harris thought initially of special, ionizing excited states. However, it is possible to realize the requirement of identical decay paths in a Λ-system with a (strong) coupling laser. The phenomenon is closely related to coherent population trapping



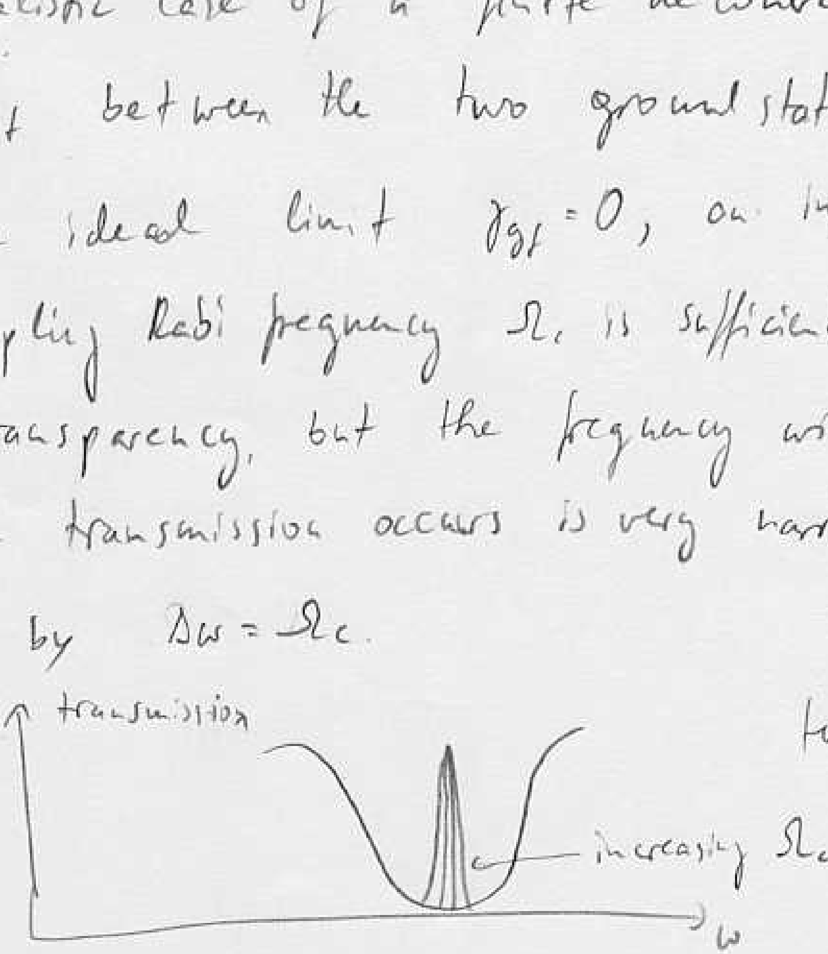
For resonant fields $\omega_1 = \omega_{ge}$, $\omega_2 = \omega_{fe}$, we have



for finite γ_{fg}

As we turn up the power of the coupling laser the transmission improves and then broadens (in the realistic case of a finite decoherence rate γ_{gf} between the two ground states.

For the ideal limit $\gamma_{gf} = 0$, an infinitesimally small coupling Rabi frequency Ω_c is sufficient to induce transparency, but the frequency window over which transmission occurs is very narrow, and given by $\Delta\omega = \Omega_c$.



for $\gamma_{fg} = 0$