Midterm Quiz

1. Hyperfine structure for positronium (13 points)

Positronium is a hydrogenic atom consisting of a positron (positive electron) and an electron.

a. (2 points) What are the quantum numbers of the energy states at low magnetic fields (only for s-states) and at high magnetic field (for arbitrary ℓ)?
b. (4 points) What are the magnetic moments (in units of the Bohr magneton) (i.e. the derivative of energy w.r.t. magnetic field) for all hyperfine states of the 1s ground state, at very low and very high magnetic fields?
c. (3 points) Sketch the energy levels vs. magnetic field. Make sure that your drawing shows correctly ALL qualitative aspects of the energy level diagram (slopes, degeneracies). Indicate the quantum numbers of the levels at low and high fields which are connected.
d. (2 points) For high magnetic fields, write down an expression for the Zeeman and hyperfine energy in terms of the quantum numbers of the energy states. You are allowed to use one unspecified pre-factor for the hyperfine energy.
e. (2 points) Are there any degeneracies at finite magnetic field? If yes, why?

2. Hydrogen atom (9 points)

Let's start from a non-relativistic description of the hydrogen atom using pointlike electron and nucleus without any spin.

a. (1 point) What are the quantum numbers and their range of values of the energy levels? Which energy levels are degenerate?
b. (2 points) Assume that the electron has a (small) finite volume. What is the effect on the energy levels (i.e. shifts, splittings)?
c. (3 points) Now assume instead that the nucleus has a spin and magnetic moment (but NOT the electron). Describe whether this leads to an additional term in the Hamiltonian and what its effect is on the energy levels and "good" quantum numbers.
d. (3 points) Now assume instead that only the electron has a spin and magnetic moment (but NOT the nucleus). What is the effect on the energy levels? In a relativistic description, how do things change?

3. Einstein rate equations in reduced dimensions (12 points)

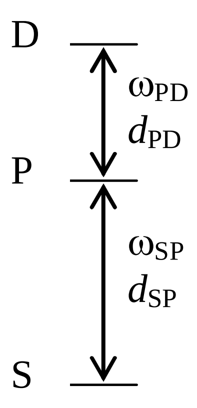
Assume that two-level atoms (excitation energy $E=\hbar\omega$) are contained in a d-dimensional box (d=1,2,3) whose walls are held at temperature T. A system with $d=1,2$ can be realized by making two or one dimensions of a three-dimension box sufficiently small.

a. (1 point) What is the relation between the atomic ground-state and excited-state populations?
b. (6 points) What is the spectral energy density of the radiation field for d=1,2,3?
c. (5 points) Derive relations between the Einstein A and B coefficients for d=1,2,3.

4. Magic-wavelength trap for a three-level atom (10 points)

Assume that an atom is well approximated by a three-level system $|S\rangle$, $|P\rangle$, $|D\rangle$ with transition frequencies ω_{SP}, ω_{PD} and transition dipole moments d_{SP}, d_{PD} as indicated in the figure. Using the AC Stark effect, an atom is to be trapped in laser beam of frequency ω.

a. (3 points) Write down the AC Stark shift for the $|S\rangle$ state in terms of the dipole moment, the frequencies and an overall constant that you need not specify. Do not assume that the rotating-wave approximation is valid.
b. (4 points) Write down the AC Stark shift for the $|P\rangle$ state as in a).
c. (3 points) For high-precision spectroscopy it is desirable that the $S\leftrightarrow P$ transition frequency be independent of trap power (magic wavelength). Determine the trap frequency ω to be used if $\omega_{SP} = 2\omega_{PD}$ and $d_{SP} = 2d_{PD}$.



5. Transfer of atomic population for a spin ½ system (6 points)

You are given a spin ½ particle, and a setup to generate constant and/or time-varying magnetic fields in the z direction and xy-plane. You can prepare the particle in an eigenstate along the z axis. Describe three conceptually different ways of transferring 50% of the population to the other eigenstate. For each method, specify the time dependence of the various fields.