8.421 Homework Assignment #1

Spring 2006, Prof. Wolfgang Ketterle

Due Friday, February 17, 2006

For questions or assistance with this assignment contact:
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1. When driven far from resonance the power dissipated in a mechanical (classical) damped oscillator increases linearly with the damping γ, but on resonance varies as γ^{-1}. Why does reducing the damping increase the power dissipated on resonance?

2. A two-state system is in the state $|1\rangle$ at t = 0. An oscillating field is applied at frequency ω with coupling matrix element $\langle 2|H'(t)|1\rangle = \hbar\omega_R \cos\omega t$. The eigenenergies of the states $|1\rangle$ and $|2\rangle$ are $\hbar\omega_1$ and $\hbar\omega_2$ respectively. Assume that $\omega_R \ll \omega_2 - \omega_1$, and $\omega_2 > \omega_1$.

a. Find the wave function for the system at $t_1 > 0$.

b. What is the probability that the system will be found in state 2 if a measurement is made at t_1?

c. The oscillating field is turned off at t_1. What is the time-dependent wave function at later times?

Hint: One can decompose the off-diagonal terms of the Hamiltonian into plane waves ($e^{\pm i\omega t}$). You are allowed to use the so-called rotating wave approximation, i.e. to neglect rapidly oscillating terms.

3. a. A spin-$\frac{1}{2}$ system at zero temperature with the magnetic moment $\overrightarrow{\mu} = \frac{-g_s\mu_0 \overrightarrow{S}}{\hbar}$ is prepared with $\overrightarrow{\mu}$ in the x-direction at t_0, and is placed in a magnetic field $\overrightarrow{B} = B_0\hat{z}$. What is the density matrix of this system? Find the expectation values of x, y, and z components of the magnetic moment for measurements made at a later time t.

Hint: Although you could solve the problem classically, you are supposed to use the full quantum treatment for the wave function and the quantum-mechanical operators for the spin components.

b. Same problem, but add longitudinal (diagonal) and transverse (off-diagonal) damping rates $\gamma_1 = T_1^{-1}$ and $\gamma_2 = T_2^{-1}$ for the spin.