Homework Assignment #2
Physics 8.421, Spring 2006, Prof. W. Ketterle
Due Wednesday, Feb. 24, 2006

1. Atomic Units

Let the atomic unit of E field be $E_A \equiv e/a_0^2$, the field of the ground state electron at the site of the proton in hydrogen.

a) On the scale of atomic units, the energy of the electrostatic potential balences the energy of quantum confinement. Use this equality to derive the atomic size, a_0. (Ignore numerical factors.)

b) Find the magnetic field of the electron at the proton, B_N. (Assume a classical orbit for the electron. If factors of 2 arise, ignore them.)

c) Find the magnetic field, B_H, which has an interaction energy of one Hartree with a Bohr magneton.

d) Express these fields in terms of E_A (Gaussian units).

e) Are there strong reasons to prefer B_N or B_H as the atomic unit of magnetic field?

2. Planck Units

a) Universal or Planck units are defined in terms of c, $\hbar$, and G which define the units of mass, length, and time. Find expressions for m_P, l_P, and t_P in terms of c, $\hbar$, and G and calculate their magnitude in SI units. These are known as the Planck mass (or Planck energy if $c \equiv 1$), Planck length, and Planck time.

b) What energies balance on the Planck scale?

3. Determination of the fine structure constant, α

a) Suppose it were possible to precisely measure the mass difference, ΔM, of two nuclear energy levels in atomic mass units (the scale where ^{12}C has a mass of 12 amu, not the atomic unit of mass, m_e), as well as the wavelength, λ (in meters), of the gamma ray emitted in the transition between them. Show that this determines the product of the fundamental constants, $N_A\hbar$. (Hint: What is Δm, the mass difference in grams?)

b) If the molar Planck constant, $N_A\hbar$, were measured accurately, this could lead to a better value for the fine structure constant, α. Find a relationship between $N_A\hbar$, α, and other accurately measured quantities. (Hint: Compare the ratio of α^2 and the Rydberg constant, R_∞, which is the Rydberg energy (half of a Hartree) divided by hc making it an inverse length.)

Current mass measurements at MIT using Dave Pritchard's single ion trap have a part-per-billion accuracy. Combined with other precision measurements, this could yield a new value of $N_A h$ and subsequently α at the few ppb level. So far, the best measurements of α are 4 to 40 ppb, and they disagree. See M.P. Bradley et al., Phys. Rev. Lett. **83**, 4510 (1999) for the latest determination of α using the mass of Cs.

TABLE I. Summary of the principal auxiliary constants used in the 1986 least-squares adjustment.

Quantity	Value	Relative uncertainty (parts in 10^9)
c	299 792 458 m/s	(exact)
V_{76-BI}	(483 594.0 GHz)($h/2e$)	(exact)
m_p/m_e	1 836.152 701(37)	20
M_p	0.001 007 276 470(12) kg/mol	12
$1+m_e/m_p$	1.000 544 617 013(11)	0.011
$1+m_e/m_d$	1.000 272 443 707(6)	0.006
$1+m_e/m_\alpha$	1.000 137 093 354(3)	0.003
$1+m_e/m_\mu$	1.004 836 332 18(71)	0.71
R_∞	10 973 731.534(13) m^{-1}	1.2
$g_e/2=\mu_e/\mu_B$	1.001 159 652 193(10)	0.010
$g_\mu/2=1+a_\mu$	1.001 165 923 0(84)	8.4
μ_e/μ_p	658.210 688 1(66)	10
μ_p/μ_B	0.001 521 032 202(15)	10
μ'_p/μ_B	0.001 520 993 129(17)	11
$d\Omega_{69-BI}/dt$	$-0.056\,6(15)$ $\mu\Omega$/a	

Rev. Mod. Phys., Vol. 59, No. 4, October 1987

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Office hours will be on Thursday, 7:00pm in 26-201, or by e-mail request.