8.421 Homework Assignment #6

Spring 2006, Prof. Wolfgang Ketterle

Due Friday, April 7, 2006

For questions or assistance with this assignment contact:
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Office Hour: Wednesday April 5 6-8pm, Rm 26-269

1. Atom-Cavity Oscillations

In lecture the interaction of a two-level atom with a single empty mode of the radiation field was discussed for the case in which the field is in resonance with the atom: $\hbar\omega = \hbar\omega_0 = E_a - E_b$. This would occur if the atom were in an ideal cavity tuned to ω_0. The problem is to find out what would happen if the cavity were tuned to some other frequency $\omega' \neq \omega_0$.

a) Does this system have stationary states? Explain.

b) The atom is placed in the cavity in its upper state, $\mid a >$. Find the probability P_a that at a later time t it is in state $\mid a >$.

c) Explain whether or not the overall energy of the system is conserved in part b).

2. Saturation Intensity

We refer to the saturation intensity of a laser for an optical transition as the intensity (power/area) at which a monochromatic beam excites the transition at a rate equal to one half of its natural line width (i.e., half of the maximum rate: this definition is not rigorous.) Find the saturation intensity for the principal transition in sodium, 590 nm. Treat the atom as a two-level system, neglecting fine and hyperfine structure. Take the natural lifetime of the 3P state to be 16 ns.

3. Atom-Cavity Oscillations for a Rydberg Atom

A Rydberg atom is an atom with an electron in a high-n state. The frequency for the transition n $\rightarrow$ n-1 is n^{-3}, and the electric dipole matrix element for the transition is n^2. (The units are atomic, and the relations approximate.)

a) Find the frequency, in Hz, of the transition n=50$\rightarrow$n=49, and the dimensions of the smallest cubical cavity which can be tuned to that resonance. (Frequency in atomic units is always in radians/time.)

b) Find the vacuum Rabi oscillation frequency for this system. Express your answer in Hz.

4. Optical Transitions Driven by Blackbody Radiation

Consider a cavity filled with blackbody radiation at a temparature T.

a) Find the average radiation energy density and the average number of photons per unit volume.

b) Find the rms E and B fields inside the cavity. What is E_{rms} inside this room?

c) Find the intensity of radiation escaping from a small hole in the wall of the cavity. (Just do the angular integral: you should get Stefan's law.)

d) The sun subtends 0.51 degrees from the earth, and the solar intensity at the earth is 1.4 kW/m^2. Find the blackbody temperature of the sun.

e) Do you have to worry about blackbody radiation when you trap a Bose-Einstein condensate in a magnetic trap? Any transition to another state will spin-flip the atoms and give the atoms recoil energy, causing the atom to be ejected from the trap. Assume that you want to ensure a trapping time of one minute. What is the maximum blackbody temperature one can tolerate assuming that the dominant electronic excitation has a transition wavelength of 590 nm and a lifetime of 16 ns?

$\int_0^\infty x^2(e^x - 1)^{-1}dx = 2.404 \qquad \int_0^\infty x^3(e^x - 1)^{-1} = \pi^4/15$

5. **RF Transition Lifetimes and RF Blackbody Transitions**

a) Estimate the lifetime of the hydrogen in the F=1 hyperfine level of the 1s state. The decay of F=1 to F=0 gives rise to the famous 21 cm line of radio astronomy. Assume that the matrix element is μ_B in your estimate.

b) A hydrogen Bose-Einstein condensate has been created in the F=1 state at MIT in the group of Dan Kleppner and Tom Greytak. At what rate does the blackbody radiation (at 4K and 300K) induce transitions to the F=0 state, which cannot be magnetically trapped?