8.421 Spring 2006 Assignment #8 Professor W. Ketterle
Due Friday, April 21, 2004

1. **Line Shape Due to a Fluctuation Field**
   This problem is somewhat artificial, but it provides an exercise in evaluating a correlation function and bears on related non-resonance phenomena such as relaxation in a fluctuation field.
   A two level system, with states $|a\rangle$ and $|b\rangle$ that have energy separation $\hbar\omega_0$, is subject to an oscillating perturbation, for instance the field of laser light, with a matrix element of the form
   $$\langle b|V(t)|a\rangle = \frac{x}{2}e^{-i\omega t}.$$
   The two levels decay with a decay rate $\gamma/2$. Furthermore, the field amplitude x is not constant but flips between two values, $+A$ and $-A$. The flipping is random, occurring with a mean rate Γ. The problem is to find the resonance line shape. This requires finding the correlation function $G_{ba}(\tau)$.
   as a guide, consider the probabilities $p_+(\tau)$ and $p_-(\tau)$ which represents the probability that if x is $+A$ at $t = 0$ it will be $+A$ or $-A$ at $t = \tau$, respectively. The correlation function can be expressed in terms of $p_+(\tau)$ and $p_-(\tau)$. From the coupled rate equations for $p_+(\tau)$ and $p_-(\tau)$, $G_{ab}(\tau)$ can be found, and from this the line shape.

2. **Convolution of Line Shapes**
   When two separate physical processes both contribute to the line shape, the resultant line shape is the convolution of the distributions. Say $D_1(\omega - \omega_0)$ and $D_2(\omega - \omega_0)$ are the normalized line shapes of the first and second type of process. The resultant line shape is then their convolution
   $$D_R(\omega - \omega_0) = \int_{-\infty}^{\infty} d\omega' D_1(\omega - \omega')D_2(\omega' - \omega_0).$$
   (a) Consider that D_1 and D_2 are both Lorentzian functions with FWHM's (full widths at half maximum) Γ_1 and Γ_2, respectively. Show that D_R is also Lorentzian and find its FWHM. (Hint: Use the Fourier convolution theorem)
   (b) Do the same if D_1 and D_2 are Gaussian and Γ_1 and Γ_2 are the r.m.s. deviations.
   (c) Find the FWHM of a Gaussian with r.m.s deviation $\Gamma/2$. Find the r.m.s. deviation of a Lorentzian with FWHM Γ.

3. **Intensity Distribution Due to Spontaneous Emission**

An atom of total angular momentum J has a spontaneous radiation rate A. It radiates to a lower level with angular momentum $J' = J - 1$. The problem is to find the rates for the various allowed transitions, i.e. the fraction of the radiation that goes into each of the possible transitions $(J, m) \longrightarrow (J', m')$. The rates can be found by applying the following considerations:
1. The sum of the rates out of each state (J, m) must equal A.
2. The sum of the rates into each state (J', m') must equal $A \times \frac{2J+1}{2J'+1}$.
3. An unpolarized mixture of radiators in level J must emit equal intensities of light with each of the three polarization components.
4. The rate for a transition $(J, m) \longrightarrow (J', m')$ must be the same for $(J, -m) \longrightarrow (J', -m')$.

Find the rates for $J = 2$, $J' = 1$. Designate the transition by letters as follows:

$$a : m = 2 \longrightarrow m' = 1$$

$$b : m = 1 \longrightarrow m' = 1$$

$$c : m = 0 \longrightarrow m' = 1$$

$$d : m = 1 \longrightarrow m' = 0$$

$$e : m = 0 \longrightarrow m' = 0$$

(a) Find the rates for a though e, and present yours results on a figure.

(b) Find the rates for a though e, using the Wigner-Eckart theorem. (Clebsch-Gordan coefficients can be either be worked out from first principles, or taken from a table in one of the quantum mechanics or spectroscopy texts.)

Note: the transition rates calculated here are important in experiments involving laser excitation. Because emission and absorption are proportional, the distribution of emission rates yields the relative strengths of exciting each of transitions.

For questions or assistance with this assignment, contact Rob Clark (robclark@mit.edu). Office hours are 1-3pm in 26-201 on Thursday, April 20, or by appointment.