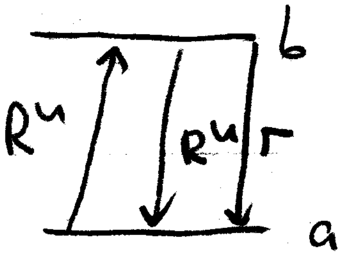
Saturation



R^u unsaturated rate

$$\dot{n}_b = -n_b(R^u + \Gamma) + R^u n_a$$
$$\dot{n}_a = n_b(\Gamma + R^u) - R^u n_a$$

Saturated rate R^S : net transfer $a \to b$ by e.m field per atom

$$R^S(n_a + n_b) = R^u(n_a - n_b)$$

Steady state $\dfrac{n_b}{n_a} = \dfrac{R^u}{\Gamma + R^u}$

$$\Rightarrow R^S = \frac{\Gamma}{2}\frac{S}{1+S}$$

$S \equiv 2R^u/\Gamma$ Saturation parameter

$R^S = R^u$ for $R^u \ll \Gamma$
$R^S \to \Gamma/2$ for $S \to \infty$

Monochromatic radiation

$$R^u = W_{ab} = \frac{\pi}{2} w_R^2 \underbrace{f(\omega)}$$

normalized line shape
Lorentzian

$$\Rightarrow R^u = \frac{w_R^2}{\Gamma} \frac{1}{1+(2\delta/\Gamma)^2}$$

$$S_{res} = 2 w_R^2 / \Gamma^2$$

$$R^s = \frac{w_R^2}{\Gamma} \frac{1}{1+(2\delta/\Gamma)^2 + 2w_R^2/\Gamma^2}$$

! important

Lorentzian with

$$\delta_{FWHM} = \frac{\Gamma}{2}\sqrt{1+S_{res}}$$

Saturation (or power) broadening

$$S = S_{res} / \left(1+(2\delta/\Gamma)^2\right)$$

$$R^s = \frac{\Gamma}{2}\frac{S}{S+1}$$

$$W_{ab} = \underbrace{\sigma_{abs}^u \frac{1}{1+S}} \frac{I_0}{\hbar\omega}$$

$$\sigma_{abs}^s \xrightarrow[S\to\infty]{} 0$$

transition bleaches out

Saturation Intensity

$$S_{res} = 1 \quad \text{for} \quad R^{u} = \Gamma/2 = \sigma_{abs} \frac{I_{SAT}}{\hbar\omega}$$

$$S_{res} = \frac{I}{I_{sat}}$$

$$I_{sat} = \frac{\hbar\omega^3}{12\pi c^2} \cdot \Gamma$$

6mW/cm² for Na D line

$$\omega_R^2 = \frac{\Gamma^2}{2} \frac{I}{I_{sat}}$$

Broadband case

$$W_{a\to b} = \sigma_0 \frac{\pi}{2} \underbrace{f(\omega_{ac})\, I}_{=\bar{I}\ \text{spectral density}} \frac{1}{\hbar\omega} \cdot \Gamma \doteq \Gamma/2 \quad \text{for } \frac{\bar{I}}{\bar{I}_{SAT}} \cdot$$

$$\Rightarrow \bar{I}_{SAT} = \frac{\hbar\omega^3}{6\pi^2 c^2}$$

indep. of details of the transition

Visible: $\bar{I}_{SAT} \simeq \frac{12W}{cm^2} \frac{1}{cm^{-1}}$