Index of refraction

$$n_r = \sqrt{1 + 4\pi\, n_{at}\, \alpha} \approx 1 + 2\pi\, n_{at}\, \alpha$$

$$\alpha = -\frac{e^2}{\hbar} |\langle k|z|g\rangle|^2 \frac{1}{\delta}$$

$\uparrow$ neglecting counterrotating term, single strong resonance, $\delta = \omega - \omega_{kg}$

Def. $\Gamma \equiv \frac{4}{3}\frac{\omega^3 e^2}{\hbar c^3} |\langle k|z|g\rangle|^2$ natural linewidth

$$\Rightarrow \quad n_r = 1 - 2\pi\, n_{at} \frac{3\Gamma}{4} \underbrace{\left(\frac{c^3}{\omega^3}\right)}_{= \frac{\lambda^3}{(2\pi)^3}} \frac{1}{\delta}$$

$$= 1 - n_{at}\, \sigma_0 \frac{\lambda}{4\pi} \frac{\Gamma}{2\delta}$$

with def. $\sigma_0 \equiv 6\pi \left(\frac{\lambda}{2\pi}\right)^2$ resonant cross section

Damping: now replace $\delta \rightarrow \delta + i\gamma/2$

Def. $\delta' = \delta/(\gamma/2)$

$$-\frac{\Gamma}{2\delta} \rightarrow \frac{\Gamma}{\gamma}\left(\frac{-1}{\delta' + i}\right) = \frac{\Gamma}{\gamma}\left[\frac{i}{1+\delta'^2} - \frac{\delta'}{1+\delta'^2}\right]$$

$$e^{ikz} = e^{i n_r \frac{\omega}{c} z} = e^{-\frac{\tilde{D}_0}{2}\left[\frac{1}{1+\delta'^2} + \frac{i\delta'}{1+\delta'^2}\right]}$$

Dispersion Absorption

optical density on resonance ($\delta' = 0$) : $\tilde{D}_0 = n_{at}\, \sigma_0 \frac{\Gamma}{\gamma} z$

Note: When linewidth γ determined by spont. em $k \rightarrow g$, then $\gamma = \Gamma$

Phase shift ϕ: $e^{ikz} = e^{-\frac{\tilde{D}_0}{2}\frac{1}{1+\delta'^2}} e^{i\phi}$

$\phi = -\frac{\tilde{D}_0}{2}\frac{\delta'}{1+\delta'^2}$; at $\delta' = \pm 1$: max phase shift $\phi = \mp \frac{\tilde{D}_0}{4}$