

## Assignment # 5

### Problem 3:

#### Rayleigh and Thomson scattering using two different interaction Hamiltonians

Elastic light scattering takes the system from state  $|i\rangle = |a; k\varepsilon\rangle$  to state  $|f\rangle = |a; k'\varepsilon'\rangle$ , where  $a$  is the (unchanged) state of the atom, and  $k$  ( $k'$ ) and  $\varepsilon$  ( $\varepsilon'$ ) the wavevector and polarization of the incident (scattered) photon.

Calculate the transition matrix element  $T_{fi}$  (i.e. matrix element of the T matrix) in leading (non-zero) order for both Rayleigh and Thomson scattering ( $\hbar\omega$  smaller/larger than excitation energies), for both the electric-dipole and the Coulomb-gauge interaction Hamiltonian  $-(q/m)pA(0) + (q^2/(2m))A^2(0)$ .

You will realize that Rayleigh scattering is more easily calculated with the dipole Hamiltonian, whereas the reverse is true for Thomson scattering. Of course, both Hamiltonians lead to the same answer.

Show that the total cross section for Thomson scattering is  $(8\pi/3)r_0^2$  where  $r_0$  is the classical electron radius.

Try to advance with this problem as far as you can by yourself. If you get stuck, you will find a discussion of this problem in Exercises 3 and 4 in Atom-Photon Interactions.