MIT 8.422 Atomic Physics II *Spring, 2009*
Prof. Wolfgang Ketterle March 11, 2009
Prof. Isaac Chuang

Assignment #5

Due: Wednesday, March 18, 2009
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Office Hours: March 16th (Mon) & March 17th (Tue), 6pm - 8pm.

1. **Unravelings of spontaneous emission dynamics**.

   The dynammics of an atom interacting with the vacuum can be desribed not just by the usual optical Bloch equations, but also, by an infinite variety of alternate pictures. We explore such *unravelings* in this problem.

   Let X, Y, and Z denote the Pauli matrices, as usual. Recall that the optical Bloch equations

$$\dot{\rho} = -\frac{i}{\hbar}[H, \rho] + \left[L\rho L^\dagger - \frac{1}{2}(L^\dagger L\rho + \rho L^\dagger L) \right] \tag{1}$$

   can be written as

$$\dot{\vec{r}} = \begin{bmatrix} -\Gamma/2 & 0 & 0 \\ 0 & -\Gamma/2 & 0 \\ 0 & 0 & -\Gamma \end{bmatrix} \vec{r} + \begin{bmatrix} 0 \\ 0 \\ \Gamma \end{bmatrix} , \tag{2}$$

   using

$$\rho = \frac{I + \vec{r} \cdot \vec{\sigma}}{2} , \tag{3}$$

   for $H = I$ (no classical driving excitation) and jump operator $L = \sqrt{\Gamma}\sigma_- = \sqrt{\Gamma}(X - iY)/2$.

   a) In lecture, we saw that the same dynamics are obtained with the same form of master equation, but with $H = -\Gamma Y/4$ and $L = \sqrt{\Gamma}(I + X - iY)/2$, instead. Prove that the dynamics are exactly the same, by writing Eq.(1) as a set of differential equations for $\vec{r}$, with this choice of H and L.

   b) Create your own alternate unraveling of the dynamics of spontaneous emission, by giving H and L (or multiple L_k) which also lead to the same differential equations for $\vec{r}$ (but are different choices than the two above). Explain the origins of your unraveling, prove its equivalence to the optical Bloch equations, and describe how your unraveling could arise in a physical situation.

2. **Two-bit code for spontaneous emission errors**. Amplitude damping is an important process in real physical systems; it models spontaneous emission, inelastic scattering, thermalization of spins to the lattice, and many other microscopic processes where energy is exchanged between the system and environment. In this problem, we study a quantum error detection code adapted for this error mechanism.

Recall that amplitude damping (spontaneous emission) for a single two-level atom is described by $\mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$, where the operation elements are

$$E_0 = \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{bmatrix} \qquad E_1 = \begin{bmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{bmatrix}. \tag{4}$$

Let $\gamma = 1 - e^{-t/T_1}$, where t is time and T_1 is the excited state lifetime.

(a) Let $|\psi_1\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, and $\rho_1 = \mathcal{E}(|\psi_1\rangle) = \sum_k E_k |\psi_1\rangle\langle\psi_1| E_k^\dagger$ be the density matrix obtained for the qubit after amplitude damping. Compute the fidelity of ρ_1 with respect to $|\psi_1\rangle$, $F_1(t) = F(|\psi_1\rangle, \rho_1) = \sqrt{\langle\psi_1|\rho_1|\psi_1\rangle}$ and plot as a function of t.

(b) Find the state $|\phi(t)\rangle$ which minimizes F_1 at each point in time, and plot this minimum value as a function of time.

(c) Let $|0_L\rangle = |01\rangle$ and $|1_L\rangle = |10\rangle$ be a quantum code encoding one logical qubit using two two-level atoms. Define $|\psi\rangle = a|0_L\rangle + b|1_L\rangle$. Compute the output state

$$\rho' = \mathcal{E}(|\psi\rangle) = \sum_{j,k=\{0,1\}} (E_j \otimes E_k)\, |\psi\rangle\langle\psi|\, (E_j \otimes E_k)^\dagger \tag{5}$$

which results when each atom independently undergoes spontaneous emission.

(d) Compute the fidelity $F(|\psi\rangle, \rho') = \sqrt{\langle\psi|\rho'|\psi\rangle}$ of ρ' with respect to $|\psi\rangle$, and plot as a function of t for the worst case state.

(e) Suppose we project the output state into the space orthogonal to $|00\rangle$ (say by performing a measurement of $Z \otimes Z$ to measure the total number of excited atoms), and keep only the cases when we do not obtain $|00\rangle$. What is the resulting state? What is its fidelity with respect to $|\psi\rangle$, as a function of t?

3. **Problem 3 will be posted on Monday**.