8.422 Atomic Physics II *Spring, 2009*

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Solutions to Assignment #8

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1. **A Zeeman Slower**

a) Maximum Deceleration

$$F = ma = \hbar k \Gamma_s = \hbar k \frac{\gamma}{2} \frac{s}{1+s}$$

as $s = I/I_{sat} \to \infty$

$$\boxed{a_{max} = \frac{\hbar k \gamma}{2m} = 9.2 \times 10^5 \, m/s^2}$$

b) Magnetic Field Profile

Since the atomic flux is proportional to v_x (see Metcalf and van der Straten *Laser Cooling and Trapping* p62), the velocity distribution is

$$f(v_x) = \frac{v_x^3}{2\tilde{v}^4} \exp\left(-\frac{v_x^2}{2\tilde{v}^2}\right)$$

and $\partial f / \partial v_x = 0$ gives you

$$v_{peak} = \sqrt{\frac{3k_B T}{m}} = 800 \, m/s$$

For a constant deceleration $a = f a_{max}$, we need to maintain a constant detuning (as seen by the atom)

$$\delta = \delta_0 + kv(x) - \mu g B(x)$$

where δ_0 is the laser detuning from the atomic resonance (with atom at rest). The velocity of something undergoing constant deceleration is

$$v(x) = \sqrt{v_0^2 - 2ax}$$

$v(L) = 0$ gives

$$\boxed{L = \frac{v_{peak}^2}{2 f a_{max}} = \frac{1}{f} \, 35cm}$$

and the magnetic field profile which maintains a constant detuning is

$$\boxed{B(x) = \frac{k}{\mu g} \sqrt{v_{peak}^2 - 2 f a_{max} \, x} + B_0}$$

where B_0 is some constant bias, and $\frac{k v_{peak}}{\mu g} = 975 \, gauss$

c) Momentum Diffusion

The scattering is described by Poisson statistics, for n scattering events in time t

$$P(n,t) = \frac{(\Gamma_s\, t)^n}{n!} e^{-\Gamma_s t}$$

for which $\langle n \rangle = \Gamma_s t$, and $\langle \Delta n^2 \rangle = \Gamma_s t$. For the absorption of photons

$$\mathcal{D}^{abs} = \frac{1}{2}\frac{d}{dt}\langle \Delta p^2 \rangle = \frac{1}{2}(\hbar k)^2 \Gamma_s$$

i. $\pm\hat{x}$ emission

$$\mathcal{D}^{emis} = \frac{\Gamma_s}{2}\left(\frac{1}{2}(\hbar k)^2 + \frac{1}{2}(-\hbar k)^2\right) = \frac{\Gamma_s}{2}(\hbar k)^2$$

$$\boxed{\mathcal{D}^{tot} = \mathcal{D}^{abs} + \mathcal{D}^{emis} = \Gamma_s(\hbar k)^2}$$

ii. isotropic emission

$$\frac{1}{4\pi}\int d\theta d\phi\, \sin\theta = 1$$

$$\mathcal{D}^{emis} = \frac{\Gamma_s}{2}\frac{1}{4\pi}\int d\theta d\phi\, \sin\theta (\hbar k \sin\theta\cos\phi)^2 = \frac{1}{6}\Gamma_s(\hbar k)^2$$

$$\boxed{\mathcal{D} = \frac{2}{3}\Gamma_s(\hbar k)^2}$$

iii. dipole emission

The probability that a photon is emitted in a solid angle at θ is proportional to $\sin^2\theta$, and $p_x = \hbar k \sin\theta\cos\phi$.

$$\frac{3}{8\pi}\int d\theta d\phi\, \sin\theta\, \sin^2\theta = 1$$

$$\mathcal{D}^{emis} = \frac{\Gamma_s}{2}\frac{3}{8\pi}\int d\theta d\phi\, \sin\theta\, \sin^2\theta(\hbar k\sin\theta\cos\phi)^2 = \frac{1}{5}\Gamma_s(\hbar k)^2$$

$$\boxed{\mathcal{D} = \frac{7}{10}\Gamma_s(\hbar k)^2}$$

2. **Slowing an Atom with Off-Resonant Light**

a) How long?

$$a = \frac{dv}{dt} = a_{max}\frac{s_0}{1 + s_0 + (2kv/\gamma)^2}$$

$$\int_{v_0}^{0} dv\left(1 + s_0 + (2kv/\gamma)^2\right) = \int_{0}^{t_{stop}} dt\, a_{max}\, s_o$$

$$\boxed{t_{stop} = \frac{1}{a_{max}s_0}\left((1+s_0)v_{peak} + \frac{4k^2}{3\gamma^2}v_{peak}^3\right) = 4.3\, sec}$$

b) How far?

$$a = \frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v$$

$$\int_0^{x_{stop}} dx\, a_{max}\, s_0 = \int_{v_0}^{0} dv\, v \left(1 + s_0 + (2kv/\gamma)^2\right)$$

$$\boxed{x_{stop} = \frac{1}{a_{max} s_0} \left(\frac{1}{2} v_{peak}^2 + \frac{k^2}{\gamma^2} v_{peak}^4\right) = 2.6km}$$

Clearly it is much more practical to use a Zeeman slower.

3. **Classical Molasses**

a) capture velocity

Start with eq. 10 in the W.D. Phillips paper. Taylor expand for large v (i.e., small 1/v):

$$F = -\frac{\hbar \gamma^3 \delta s_0}{2k^2}\, v^{-3} = -\alpha\, v^{-3}$$

Integrate just as in 2(b):

$$\int_0^{2r} dx\, (-\alpha/m) = \int_{v_{cap}}^{0} dv\, v^4$$

$$\boxed{v_{cap} = \left(\frac{10\alpha}{m} r\right)^{1/5} = 12.5\, m/s}$$

b) maximize diffusion time

The answer is given in ch 6 of the W.D. Phillips paper. The optimal conditions are $\delta = -\gamma/2$ and $s_0 = 1/6$. This gives an optimized diffusion time of

$$\boxed{t_d^{max} = \frac{4k^2 \langle r^2 \rangle}{27 N^2 \gamma} = 120\, ms}$$