MIT 8.422 Atomic Physics II *Spring, 2009*
Prof. Wolfgang Ketterle April 15, 2009
Prof. Isaac Chuang

Assignment #9

Due: Wednesday, April 22th, 2009
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Office Hours: April 20th (Mon) & April 21th (Tue), 1pm - 3pm.

THE MAGNETO-OPTICAL TRAP

1. **1-D MOT**. The motion of an atom in a one-dimensional magneto-optical trap obeys the equation of a damped harmonic oscillator:

$$\ddot{z} + \gamma\dot{z} + \omega^2 z = 0$$

In addition to the forces of the damped harmonic oscillator, the atoms feel a fluctuating force due to the spontaneous emission which is a source of heating. Typical values (for a saturation parameter $S \approx 1$) are $\gamma^{-1} = 100\mu s$ for the damping time and $\omega = 2\pi\times$ 1 kHz for the trap frequency.

Suppose we are trapping sodium atoms ($M = 23$ amu, $\lambda = 589$ nm) with a laser intensity such that the scattering rate is $10^7 s^{-1}$.

(a) Determine the equilibrium 1-D temperature using the given values for the damping and scattering rates.

(b) What is the size of the trapped cloud at this temperature?

2. **Density Limit in a MOT**. In a 3-D magneto-optical trap, the density of the trapped atoms is limited by a net outward radiation pressure which opposes the trapping force. We can divide the density-dependent photon-pressure force into two parts. First there is a repulsive 'radiation trapping force' due to atoms absorbing photons scattered from other atoms in the trap. Also, there is an attractive 'attenuation force' which is caused by atoms at the side of the cloud attenuating the laser beams, thus creating an intensity imbalance which leads to an inward force.

(a) Show that the radiation trapping force obeys the equation

$$\nabla \cdot \mathbf{F}_R = \frac{6\sigma_L\sigma_R I n}{c},$$

where $\mathbf{I}$ is the intensity of one of the trapping laser beams, n is the number density of atoms in the cloud, σ_L is the cross-section for absorption of the laser beam, and σ_R is the cross-section for absorption of the scattered light. (**Hint**: Find the magnitude of the force between two atoms separated by a distance d, where one atom re-radiates a laser photon and the second atom absorbs it. Now, since this is an inverse-square force, you can use Gauss' law to find $\nabla \cdot \mathbf{F}_R$. Assume that photons are only scattered twice.)

(b) The attenuation force may be obtained simply by replacing σ_R with $-\sigma_L$, so that

$$\nabla \cdot \mathbf{F}_A = -\frac{6\sigma_L^2 I n}{c}.$$

Explain why this is so.

(c) The total force is the sum of $\mathbf{F}_R, \mathbf{F}_A$, and the trapping force $\mathbf{F}_T = -\kappa \mathbf{r}$, where κ is the spring constant of the trap. For stability we require that the total force is attractive, $\nabla \cdot \mathbf{F}_{total} < 0$. Find the maximum density of the trapped cloud at a given κ from the condition $\nabla \cdot \mathbf{F}_{total} = 0$.

(d) Give a qualitative argument for whether we expect $\sigma_R = \sigma_L, \sigma_R > \sigma_L$, or $\sigma_R < \sigma_L$. (**Hint**: sketch the absorption and emission spectra for an atom in a strong laser field with a red detuning.)

(e) Suppose that some of the atoms can be put into a 'dark state', so that only a fraction f of the atoms absorb the trapping light. How do F_R, F_A and F_T vary with f? What happens to the limiting density n_{max}? This is the concept of the Dark SPOT trap (PRL **70**, 2253 (1993)).

3. Optical dipole trap When atoms are loaded into a far-off resonant dipole trap, they are accelerated by the stimulated light force. In the absence of an spontaneous emission, where does the kinetic energy come from? In this problem, we shall discuss this situation in an idealized geometry. As described in Figure 1, a finite number of atoms is initially stored in reservoir A, and they can move into reservoir B thorough a thin capillary. For simplicity, we assume non-divergent uniform electromagnetic(EM) field in an isolated region (dotted region in figure 1, total volume V). Moreover, EM field has large detuning $\delta < 0$ (no spontaneous force) so that the atoms will move into the reservoir B at $t = t_0$. Assume that the atom density increases linearly up to $\dot{n}\triangle t$ during $\triangle t$. Since the atoms shift the phase of the transmitted EM field, the frequency of EM field measured at the position D is shifted by $\triangle\omega$ in the lab frame(figure 2). In this problem, we will show that the total energy change of the atoms in the reservoir B is equal to the energy decrease of EM field accumulated between $t = t_0$ and $t = t_0 + \Delta t$.

a) First, let an incident EM field be $\overrightarrow{E_i} \propto \hat{x}e^{i(wt-kz)}$. When some atoms are introduced in the reservoir B, the index of refraction is changed: $n_r \simeq 1 + \frac{1}{2}n(t)\alpha(w)$ where $\alpha(w)$ is the polarizability. By considering the phase shift due to the atoms in the reservoir B, calculate a frequency shift $\Delta\omega$ of the EM field at the position D. (see figure 2) You will get $\overrightarrow{E_D} \propto \hat{x}e^{i([w-\Delta w]t-kz)}$ when $t_0 < t < t_0 + \Delta t$.

b) With the result of (a) $\Delta\omega$, calculate the total energy decrease of the EM field, ΔE_{EM}, accumulated between $t = t_0$ and $t = t_0 + \Delta t$.

c) By considering Stark effect, we can calculate the polarizability $\alpha(w)$. Given the $\alpha(w)$, calculate the energy shift, $\Delta E_{Stark}|_{t=t_o+\Delta t}$, due to Stark effect. Is ΔE_{Stark} equal to ΔE_{EM}? Why? (or Why not?)

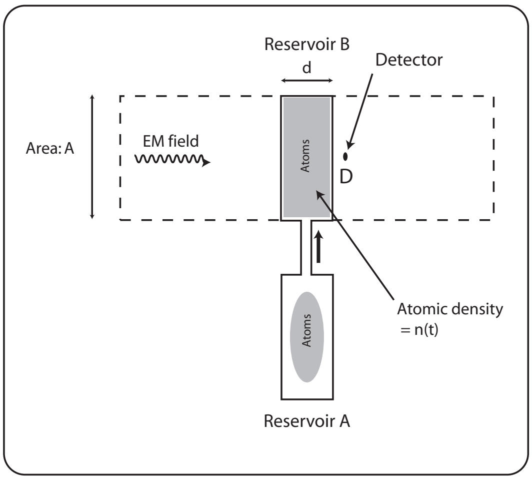

Figure 1

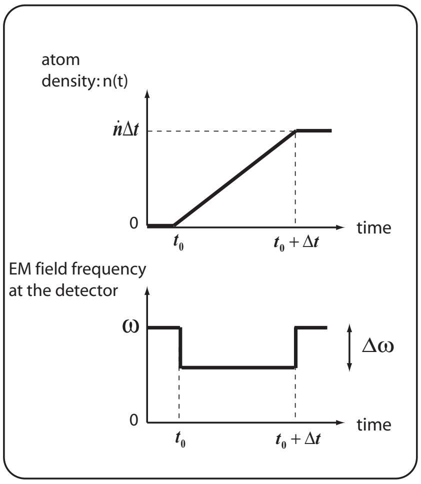

Figure 2