Phase-coherent amplification of atomic matter waves

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Atomic matter waves, just like electro-magnetic waves, can be focussed, reflected, guided, and split by the passive atom optical elements of today. However, the key for many applications of RF and light waves lies in the availability of amplifiers. These active devices allow small signal detection and have led to the development of masers and lasers. Here we report the observation of phase coherent amplification of atomic matter waves. The active medium was a Bose-Einstein condensate pumped by far off-resonant light. An atomic wave packet was split off the condensate by diffraction from an optical standing wave and then amplified. The phase coherence of the amplifier was verified by interfering the output with a reference wave packet. This development provides a new tool for atom optics and atom interferometry and opens the door to active matter wave devices.

Matter wave amplification differs from light amplification in one important aspect. Since the total number of atoms is conserved (in contrast to photons), the active medium of a matter wave amplifier has to include a reservoir of atoms. One also needs a coupling mechanism which transfers atoms from the reservoir to the input mode while conserving energy and momentum. Amplification is realized if the transfer mechanism is accelerated by the build-up of atoms in the final state and inversion is maintained between the initial and final states.

It is convenient to use a Bose-Einstein condensate with its high phase-space density to realize matter wave amplification, although in principle it is not necessary[1-4]. This is analogous to the situation in optics where coherent light is used for pumping parametric amplifiers. The momentum required to transfer atoms from the condensate at rest to the input mode can be provided by light scattering. As was discussed in [5,6], a condensate pumped by an off-resonant laser acts as a matter wave amplifier. It can amplify input matter waves within the momentum range which can be reached by scattering a single pump photon. Energy is conserved because the scattered light has lower frequency.

The inversion in this matter wave amplifier is most apparent in the dressed atom picture, where the condensate at rest and the pump light field form an upper state that can decay into recoiling atoms and scattered photons. The escape of the photons maintains inversion allowing, in principle, a complete transfer of the condensate atoms into the recoil mode.

The gain process can be explained in a semiclassical picture. The input matter wave of wave vector $\boldsymbol{K}_j$ interferes with the condensate at rest and forms a moving matter wave grating which diffracts the pump light with wave vector $\boldsymbol{k}_0$ into the momentum and energy conserving direction $\boldsymbol{k}_i = (\boldsymbol{k}_0 - \boldsymbol{K}_j)$. The momentum imparted by the photon scattering is absorbed by the matter wave grating by coherently transferring an atom from the condensate into the recoil mode, which is the input mode. The diffraction efficiency of the grating is proportional to the square of the depth of the density modulation, and therefore to the number of atoms in the input mode N_j. This implies an exponential growth of N_j (as long as one can neglect the depletion of the condensate at rest).

The amplification of atoms in a recoil mode j follows a gain equation [7,8]

$$\dot{N}_j = (G_j - L_j)N_j \tag{1}$$

with the gain coefficient

$$G_j = R\,N_0\,\frac{\sin^2\theta_j}{8\pi/3}\Omega_j. \tag{2}$$

Here R is the rate for single-atom Rayleigh scattering which is proportional to the pump light intensity, N_0 the number of atoms in the condensate at rest, θ_j the angle between the polarization of the incident light and the direction of the scattered light, and Ω_j the phase-matching solid angle for scattering into mode j. The loss term L_j describes the decoherence rate of the matter-wave grating and determines the threshold for exponential growth (see [7] for details). We have recently observed the build up of superradiant emission due to this gain and confirmed the salient features of the model presented above. Previous theoretical discussions on this system had assumed an optical cavity into which the pump light is scattered [5,6], but our experiment [7] showed that cigar-shaped condensates have sufficient gain for axial light emission, even in free space. In our observation of superradiance, there were no input atoms, and the build-up of a macroscopic matter wave was initiated by spontaneous scattering. Here we characterize the amplification process by providing a variable input and measuring the amplitude and phase of the amplified matter wave.

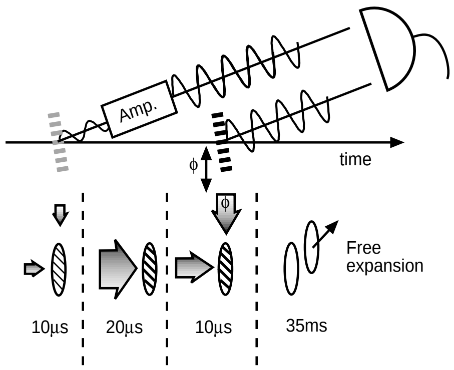

FIG. 1. Experimental scheme for observing phase coherent matter wave amplification. A small-amplitude matter wave was split off the condensate by applying a pulse of two off-resonant laser beams (Bragg pulse). This input matter wave was amplified by passing it through the condensate pumped by a laser beam. The coherence of the amplified wave was verified by observing its interference with a reference matter wave, which was produced by applying a second (reference) Bragg pulse to the condensate. The interference signal was observed after 35 ms of ballistic expansion. Note that the polarization of the radial beam was perpendicular to the long axis of the condensate ($\theta = \pi/2$). Extinction of the axial beam during the amplifying pulse was better than 10^{-8}.

Cigar-shaped Bose-Einstein condensates of a few million sodium atoms in the $F = 1$, $m_F = -1$ state were produced in a magnetic trap using the standard techniques of laser cooling and evaporative cooling [9]. The condensate was 200 μm in length and 18μm in diameter. Input matter waves with a well defined momentum were generated by exposing the condensate to a pulsed optical standing wave which transferred a small fraction of the atoms ($10^{-4} \sim 10^{-2}$) into a recoil mode by Bragg diffraction [10,11]. The frequencies of both laser beams were red-detuned by 1.7 GHz from the $3S_{1/2}, F = 1 \rightarrow 3P_{3/2}, F = 0, 1, 2$ transition and the intensities were kept below the threshold for superradiant Rayleigh scattering [7]. The geometry of the light beams is shown in Fig. 1. The beam which was perpendicular to the long axis of the condensate (radial beam) was blue detuned by 50 kHz relative to the axial beam. This detuning fulfills the Bragg resonance condition, i.e. , it corresponds to the kinetic energy of the recoiling atoms. The broadening of the Bragg resonance by the finite pulse length (10 μs) was small enough to prevent the off-resonant excitation of other diffraction orders.

Amplification of the input matter wave was realized by applying an intense pump pulse along the direction of the radial Bragg beam for the next 20 μs with a typical intensity of 40 mW/cm^2. The number of atoms in the recoil mode was determined by suddenly switching off the trap and observing the ballistically expanding atoms after 35 ms of time-of-flight using resonant absorption imaging. After the expansion, the condensate and the recoiling atoms were fully separated (Fig. 2a-c).

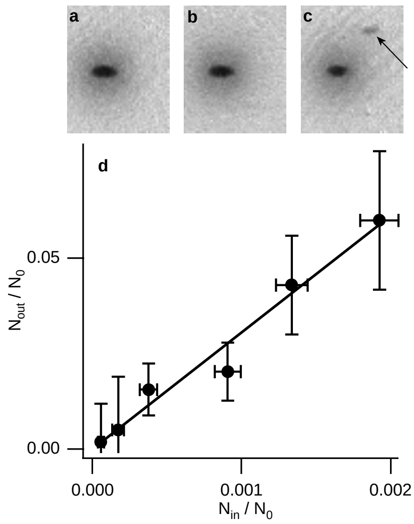

FIG. 2. Input-output characteristics of the matter wave amplifier. (**a-c**) Typical time-of-flight absorption images demonstrating matter wave amplification. The output of the seeded amplifier (**c**) is clearly visible, whereas no recoiling atoms are discernible in the case without amplification (**a**) or amplification without the input (**b**). The size of the images is 2.8 mm × 2.3 mm. (**d**) Output of the amplifier as a function of the number of atoms at the input. A straight line fit shows a number gain of 30. Note that the same data set as in Fig. 4 was used. The total number of amplified atoms was determined by measuring the increase in the offset of the obtained interference fringes discussed below.

Fig. 2 shows the input-output characteristics of the amplifier. The number of input atoms was below the detection limit of our absorption imaging (Fig. 2a) and was determined from a calibration of the Bragg process at high laser powers, where the diffracted atoms were clearly visible in the images. For short pulses, the diffraction efficiency was quadratic in the laser power, as expected. The amplification pulse alone, although above the threshold for superradiance [7], did not generate a discernible signal of atoms in the recoil mode (Fig. 2b). When the weak input matter wave was added, the amplified signal was clearly visible (Fig. 2c). The gain was controlled by the intensity of the pump pulse (see eq.(2)) and typically varied between 10 and 100. Fig. 2d shows the observed linear relationship between the atom numbers in the input and the amplified output with a number gain of 30.

The phase of the amplified matter wave was determined with an interferometric technique. For this, a reference matter wave was split off the condensate in the trap in the same manner as the first (input) wave. The phase of the reference matter wave was scanned by shifting the phase of the radio-frequency signal that drove the acousto-optic modulator generating the axial Bragg beam. We then observed the interference between the reference and the amplified matter waves by measuring the number of atoms in the recoil mode.

If we characterize the two interfering matter waves by their atom numbers N_i and normalized wavefunctions ψ_i, then the visibility V of the interference pattern is given by

$$V = \frac{2 < \psi_1|\psi_2 > \sqrt{N_1 N_2}}{N_1 + N_2} \tag{3}$$

$$\sim 2 < \psi_1|\psi_2 > \sqrt{N_1/N_2} \qquad (N_1 \ll N_2) \tag{4}$$

This simple description is valid for small transfer efficiencies of the Bragg pulses ($N_i \ll N_0$). A more general description treats the two Bragg pulses as a Ramsey type matter wave interferometer [12], where the phase of the matter wave grating (formed by the recoiling atoms and the condensate) is compared with the phase of the standing wave light grating used for the second Bragg transition. This treatment reproduces eq.(3), where N_1 is replaced by $N_1(1 - N_2/N_0)$ and N_2 by $N_2(1 - N_1/N_0)$.

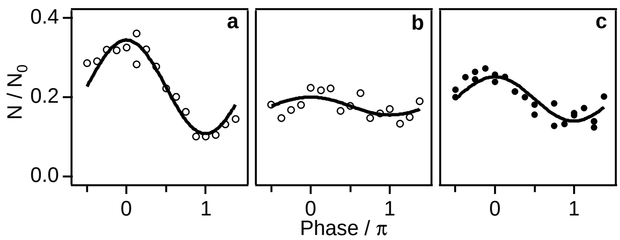

FIG. 3. Phase coherent amplification of matter waves. Shown is the number of atoms N in the recoil mode versus the relative phase between the two Bragg pulses. (**a**) Interference between two matter waves of almost equal intensity yielded a high contrast fringe (visibility $V = 52\%$). (**b**) When the intensity of the first matter wave was decreased by a factor of 40, the visibility V dropped to the noise level($< 15\%$). (**c**) By switching on the amplifier, a large visibility was regained ($V = 31\%$).

Fig. 3 shows the interference signal between the amplified input and the reference matter wave as the reference phase was scanned. When the input was comparable in intensity to the reference matter wave, high contrast fringes were observed even without amplification (Fig. 3a). Fringes were barely visible, when the input was about 40 times weaker in population (Fig. 3b). After amplification, we regained a large visibility(Fig. 3c). This increase in visibility proves the coherent nature of the matter wave amplification process.

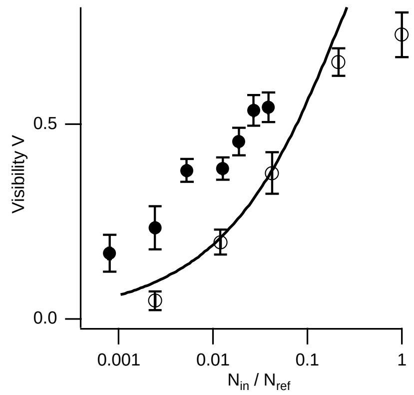

FIG. 4. Visibility of interference with and without amplification. The visibility of the interference pattern between input (output) and the reference matter wave is shown with open (filled) circles as a function of the input number of atoms N_{in} (normalized by the population of the reference matter wave N_{ref}). The solid line is the expected visibility from eq.(3), assuming an overlap factor of unity. The intensity of the amplification pulse was 40 mW/cm^2

The visibility was studied as a function of the number of input atoms. Since amplification did not produce any observable phase shift, the visibility could be measured by comparing the number of atoms in the recoil mode at constructive and destructive phases. Without amplification, the visibility increased with the number of input atoms in fair agreement with eq.(3), assuming $< \psi_1|\psi_2 > \sim 1$ for the overlap factor. With amplification (under the same conditions which gave a number gain of 30 (Fig. 2)), the visibility increased by a factor of about two over a wide range of input intensities. Since for a small number of atoms N_1, the visibility increases as $\sqrt{N_1}$ (eq.(4)), this implies a "coherent matter wave gain" of at least four. The discrepancy to the number gain of 30 may be due to a distortion of the input matter wave during its amplification which would reduce the overlap factor $\langle\psi_1|\psi_2\rangle$. Indeed, the shape of the amplified matter wave looked distorted after the 35 ms of time-of-flight. This indicates a wavefront distortion of the amplified matter wave which can be parameterized by introducing an overlap factor of $\sqrt{4/30} = 0.4$ in eq. (3), but this effect requires further study. No distortion was seen at short times (1.5 ms) after the amplification (as observed by phase-contrast imaging). This and a simple model suggest that spatially non-uniform gain is not the dominant cause for the non-ideal visibility.

The interference between the (amplified) signal and the reference matter wave was measured in a two-pulse interferometer where the delay time between the pulses was kept shorter than the the coherence time of the two-photon Bragg process (~ 40 μs [7]). The coherence time can be determined by the linewidth of the Bragg resonance [11], or by the decay rate of Ramsey fringe visibility [12]. In the absence of mean-field broadening, it is proportional to the spatial extent of the condensate divided by the recoil velocity [11,12].

We deliberately studied the matter wave amplification in the small-signal limit. We chose low density of the condensate, small number of input atoms, and rather low gain in order to avoid additional complexities such as elastic collisions and repeated superradiant scattering [7]. The density of the condensate was adjusted to be less than 2×10^{14} cm^{-3} by limiting the number of condensed atoms. The average number of (incoherent) elastic collisions occurring between the two Bragg pulses was less than 0.05 per recoiling atom and should be negligible.

The input matter waves were generated in perfect spatial overlap with the condensate by Bragg pulses. In further experiments, we have also observed the single-pass amplification of input atomic wave packets which were initially spatially separated from the condensate. This was achieved by letting the input wave packet oscillate in the trapping potential for half an oscillation period before applying the amplification light pulse. As the wave packet passed through the pumped condensate at rest, amplification was clearly observed.

The matter wave amplifier which we have demonstrated amplifies only the recoil states that can be populated by normal Rayleigh scattering. Their momentum is located on a sphere with radius $|\hbar \boldsymbol{k}_0|$ centered at the incoming photon momentum ($\hbar \boldsymbol{k}_0$). We have achieved perfect momentum matching by using the radial beam for the Bragg process as the pump beam for the amplification process. Ultimately, the momentum bandwidth of the amplification process should be given by the momentum uncertainty of the condensate, which is roughly Planck's constant divided by the size of the condensate.

The amplification process studied in this work is based on bosonic stimulation: if N bosons occupy a given state, the transition rates into that state are proportional to $(N + 1)$. Some experiments have seen such accelerated transition rates without observing phase coherence [7,13,14]. A study of the formation of the condensate showed evidence for bosonic stimulation in the elastic scattering of atoms into the condensate [13], an irreversible process establishing thermal equilibrium. The recently demonstrated four-wave mixing of atoms [14] also relied on bosonic stimulation of atom-atom collisions, but in this case as a coherent process mediated by the mean field.

Bosonic stimulation has been discussed as the gain mechanism of atom lasers, either based on evaporation or optical pumping [1-4]. In these discussions, the idea was to start with a thermal cloud of low phase-space density and create a large population in a single matter wave mode. Our experiments exploited the high phase space density of a condensate to realize a high matter wave gain. In principle, we could have used an optically pumped thermal cloud as a gain medium, but the gain would have been extremely small due to the large Doppler broadening.

Our experiment can also be regarded as a demonstration of an active atom interferometer. It realizes a two-pulse atom interferometer [15] with phase-coherent amplification in one of the arms. Such active interferometers may be advantageous for precise measurements of phase shifts in highly absorptive media, e.g. , for measurements of the index of (matter wave) refraction when a condensate passes through a gas of atoms or molecules [16]. Since the most accurate optical gyroscopes involve active interferometers [17], atom amplification might also play a role in future matter-wave gyroscopes [18].

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