

Amplification of Local Instabilities in a Bose-Einstein Condensate with Attractive Interactions

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We study the collapse of large homogeneous Bose-Einstein condensates due to intrinsic attractive interactions. We observe the amplification of a local instability by seeding a momentum state \mathbf{p} and suddenly switching the scattering length negative via a Feshbach resonance. We also observe the appearance of atoms in the conjugate momentum state as required by momentum conservation. For large condensates, the time scale for this depletion process becomes faster than that for global collapse.

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Our current understanding of the collapse of Bose-Einstein condensates (BECs) with attractive interactions is incomplete. While the experiments in ^7Li provided many insights into the formation kinetics and stability [1], it was not until the discovery of externally induced Feshbach resonances [2] that it became possible to tune the value of the scattering length and study in detail the effects of an attractive mean field potential. This technique was used by Donley *et al.* to study ^{85}Rb with a negative scattering length, and they have observed rich dynamics inherent in the collapse of the condensate. Among the most intriguing observations were the formation of low energy “bursts” which were ejected out of the condensate [3]. While theoretically the enhancement of quantum fluctuations could give rise to such phenomena [4–6], there is currently no consensus on the exact mechanism by which it occurs.

Thus far, theoretical developments have been limited by having only one experimental testing ground. The experiments using ^{85}Rb were done using small condensates ($\sim 15\,000$ atoms), where the attractive mean field energy μ is comparable to the $\hbar\omega$ level spacing of the harmonic trapping potential. In this paper, we study the collapse of large sodium condensates far in the Thomas-Fermi regime ($|\mu| \gg \hbar\omega$), where the spatial profile of the condensate is relatively homogeneous. Much of the dynamics of such a system is then described by local phenomena. When the interactions become attractive, Yurovsky [5] predicts that local instabilities with momentum on the order of the (imaginary) speed of sound will undergo exponential growth. Simultaneously, momentum conservation requires atoms to be generated in conjugate momentum states. Since amplification happens on the time scale of the chemical potential h/μ , the resulting quantum evaporation of the zero momentum condensate atoms can happen faster than the global collapse where the whole condensate “implodes.”

We probe for local instabilities by seeding a particular momentum state with an initial population, then suddenly switching the scattering length negative via a Feshbach resonance. At the same time, the trapping potential is

turned off, so all subsequent dynamics are due only to the intrinsic attractive interactions. The resulting amplification and the associated generation of atoms in the conjugate momentum state verifies the theory. We end with a discussion on the different collapse time scales of competing processes and show that, for large condensates, this decay channel becomes dominant.

The theoretical basis for the amplification of local instabilities is the dispersion relation for the elementary excitations in a Bose-Einstein condensate:

$$\epsilon(\mathbf{p}) = \sqrt{\frac{|\mathbf{p}|^2}{2m} \left(2n_0U + \frac{|\mathbf{p}|^2}{2m} \right)}, \quad (1)$$

where n_0 is the density, $U = 4\pi\hbar^2 a/m = \mu/n_0$ is the contact potential, a is the scattering length, and m is the atomic mass. For an elementary excitation whose momentum satisfies $|\mathbf{p}|^2/2m < 2|\mu|$, an instability forms when $\mu < 0$ (i.e., $a < 0$) and oscillatory behavior gives way to exponential growth or decay. A formal derivation gives the evolution of these low momentum modes as

$$\langle \xi_{\mathbf{p}}^\dagger \xi_{\mathbf{p}} \rangle(t) = \frac{|Un_0|^2}{\hbar^2 \lambda^2(\mathbf{p})} \sinh^2[\lambda(\mathbf{p})t], \quad (2)$$

where the growth rate $\lambda(\mathbf{p})$ is given by $|\epsilon(\mathbf{p})|/\hbar$ [Eq. (1)] when $a < 0$ [5] and $\xi_{\mathbf{p}}$ is the destruction operator for mode \mathbf{p} . The instability of the mode pair $(\mathbf{p}, -\mathbf{p})$ results in correlated growth, where the creation of an atom in the $+\mathbf{p}$ state is accompanied by the creation of an atom in the $-\mathbf{p}$ state. A similar phenomenon is also responsible for the four wave mixing process observed in [7,8]. At higher momentum, the energy becomes real again as the excitations now have enough kinetic energy to stabilize them against the attractive interactions.

In our experiments, we created large cigar-shaped sodium condensates in the $F = 1$, $m_F = -1$ spin state with typical atom numbers of ~ 30 million and peak densities of $3 \times 10^{14} \text{ cm}^{-3}$ in a Ioffe-Pritchard magnetic trap. Following this, they were adiabatically loaded into a mode-matched 1064 nm cylindrical optical dipole trap with trapping frequencies of 250 and 2 Hz and held for 1 s

to allow transient excitations to damp out. In the next 117 ms, the axial magnetic field was ramped up to an intermediate value just short of the 1195 G Feshbach resonance [9]. Two pairs of coils coaxial with the condensates contributed to the bias field: The bias coils produced a large field of ~ 1200 G and the pinch coils produced a small field of ~ -8 G. Since their mutual inductance was small, the small pinch coils could be switched off suddenly to jump up the field in under $10 \mu\text{s}$ with a negligible adverse effect on the bias field (≤ 1 G). By changing the intermediate field value, we were also able to tune the scattering length a continuously from the background scattering length a_0 to zero to arbitrary negative values. The reproducibility of the resulting magnetic field was 0.4 G. At this point, the condensates were slightly compressed by a small axial magnetic field curvature, giving them radial and axial dimensions of $\sim 15 \mu\text{m}$ and 1.5 mm, respectively, and a speed of sound $c = \sqrt{Un_0/m}$ of 8 mm/s.

We imprinted a low momentum excitation onto a stable condensate ($a > 0$) using a two photon optical Bragg transition to couple atoms from the zero momentum state to a low momentum state \mathbf{p} [10]. The Bragg beams were pulsed on for $400 \mu\text{s}$ and were directed at an angle of 15° and 20° , respectively, from the long axis of the condensate, creating phonons propagating with a momentum of $m \times 2.5$ mm/s in a predominantly radial direction. With this momentum \mathbf{p} , the minimum value of $|a|$ required for amplification to happen is estimated to be -0.06 nm. The beams were red-detuned from the sodium D_2 line by 3 nm to minimize Rayleigh scattering, and had a frequency difference of $\Delta\omega = 2\pi \times 700$ Hz.

At this point, we changed the scattering length a suddenly and entered the Feshbach resonance by turning off the pinch coils. This technique reduced the time available for three-body decay, thereby overcoming the problematic high loss rate first observed by Inouye *et al.* [2] and gave us a well-defined initial condition similar to that in [3]. At the same instant, we also turned off the optical trapping potential to allow the system to evolve under its intrinsic attractive interactions. After a variable time of 0 to 1 ms, the magnetic fields were switched off, causing the scattering length to become positive again. Following 6 ms of ballistic expansion, a pump pulse from $F = 1$ to $F' = 2$ was applied. Radial absorption images providing the momentum distribution of the condensates were taken on the $F = 2$ to $F' = 3$ cycling transition.

The images in Fig. 1 probe the radial dynamics of the condensates and provide dramatic visual verification of quantum evaporation. Figure 1(a) shows the $+\mathbf{p}$ excitations moving out of the condensate without any amplification. In contrast, Fig. 1(b) was taken after the condensate had been held at $a = -0.82$ nm for $600 \mu\text{s}$. Not only was the number of atoms in the $+\mathbf{p}$ momentum state significantly amplified, it was accompanied by the formation of excitations in the $-\mathbf{p}$ momentum state, here

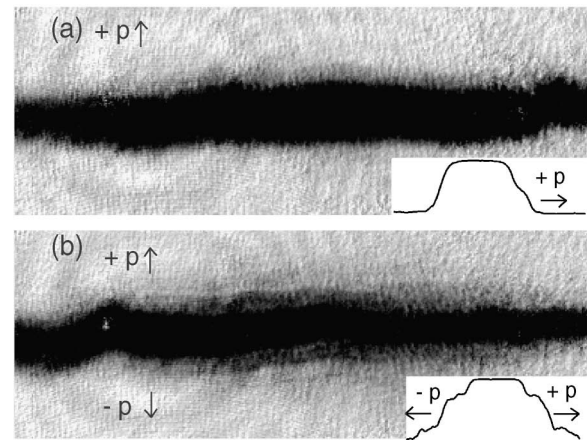


FIG. 1. Amplification of unstable excitations in a BEC with $a < 0$. In (a), the unamplified $+\mathbf{p}$ excitations are shown moving out of the condensate after 6 ms ballistic expansion. In (b), the $+\mathbf{p}$ excitations have been amplified and are accompanied by the formation of $-\mathbf{p}$ excitations after $600 \mu\text{s}$ of hold time at $a = -0.82$ nm. The field of view is $960 \mu\text{m}$ by $100 \mu\text{m}$. The insets show a radial optical density profile in arbitrary units averaged over a $\sim 100 \mu\text{m}$ long section of the condensate which was relatively uniform.

seen moving out of the condensate in the opposite direction. These observations clearly demonstrate the instability of a condensate with negative scattering length.

Because of the large aspect ratio of our condensates, only part of the condensate could be imaged at high magnification. However, since the $+\mathbf{p}$ and $-\mathbf{p}$ excitations were created predominantly in the radial direction, this was not a limitation. The small “kinks” that were also apparent in our condensate are most likely a result of imperfections in our optical dipole trap. Yet, rather than degrade the signal, they highlight the parallel contours between the condensate and the ridge of excitations as atoms move out with a definite momentum.

In order to perform more quantitative tests of this phenomenon, we first characterized the negative scattering length dependence on the field by directly probing the strength of the attractive interactions. As before, optically trapped condensates close to a Feshbach resonance were prepared. The confining infrared laser beam was then replaced with a repulsive 3 nm blue-detuned “antitrap” beam as we simultaneously jumped to negative scattering lengths. At the right intensity I , the antitrap beam provided the correct amount of repulsive dipole force needed to compensate for the attractive interactions within the condensate and suppressed any global contraction of size. However, this is an unstable equilibrium and any sloshing of the condensate or misalignment of the laser beam caused the condensate to be repelled. Therefore the antitrap was fine aligned to milliradian accuracy such that a condensate with $a > 0$ was ripped apart radially into a hollow cylinder. Using this method, we were able to stabilize an attractive condensate for 0.2 to 2 ms, depending on $a(B)$, before unavoidable losses became significant.

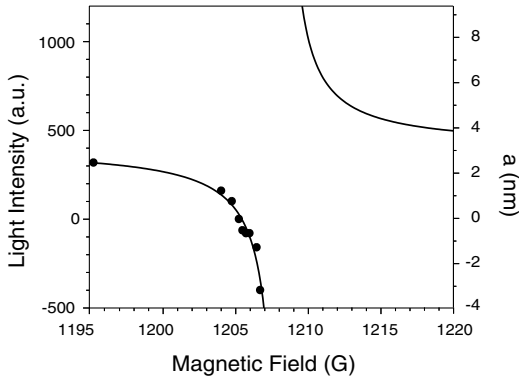


FIG. 2. Characterization of the Feshbach resonance. The light intensity needed to balance out the interatomic interactions gives the dependence of the scattering length on magnetic field. The solid line is a fit to the data.

The radial dimension was used to monitor the mechanical dynamics of the condensate occurring on the 250 Hz time scale of the trap frequency. For minimal distortion of the spatial image, absorption images were taken after only 2 ms of ballistic expansion necessary for the high magnetic fields to die out.

In equilibrium, $a \propto F_{\text{attractive}} = F_{\text{repulsive}} \propto I$, and we obtain the scattering length dependence on the magnetic field by plotting I vs B (Fig. 2). A red-detuned laser beam with a similar detuning of 3 nm was employed to obtain the points for positive a . By fitting the expected Feshbach curve $a(B) = a_0[1 + (\Delta B)/(B - B_0)]$ to our data, we find the width ΔB of the resonance to be (2.4 ± 0.4) G which determines the range of the magnetic field we have to work within [11]. Here, $a_0 = 3.3$ nm is the triplet scattering length at high fields [12].

A quantitative analysis of the growth in the $+\mathbf{p}$ and $-\mathbf{p}$ modes was performed by monitoring their occupation number as a function of hold time in the attractive regime (Fig. 3). The number count per mode was normalized to the number of atoms in the condensate at the end of the hold time to account for the high loss of atoms during this process due to inelastic collisions [13]. The maximum duration of amplification was limited by the lifetime of the condensate, which was about $600 \mu\text{s}$ for $a = -1.35$ nm. Following Eq. (2), an exponential dependence was fitted to the data, yielding a common growth rate of $\lambda = 5.89 \pm 0.83 \text{ ms}^{-1}$ for both modes. This agrees well with the theoretical value of 5.57 ms^{-1} , estimated using our initial mean field of $h \times 5$ kHz.

We also investigated the dependence of the growth rate on the strength of the interactions. By varying the scattering length and extracting λ as above, we observed a strong increase of λ with $|a|$ (Fig. 4). A fit to the theoretical prediction $\lambda(\mathbf{p}) = b_1\sqrt{|a|} - b_2$ [Eq. (1)] yielded the fit parameters $b_1 = (1.11 \pm 0.14) \times 10^8 \text{ nm}^{1/2} \text{ ms}^{-1}$ and $b_2 = 0.078 \pm 0.04$ nm, in agreement with theoretical estimates of $1.55 \times 10^8 \text{ nm}^{1/2} \text{ ms}^{-1}$ and 0.06 nm, respectively. For this sequence of measurements, the initial mean field was $\mu = h \times 4$ kHz. The

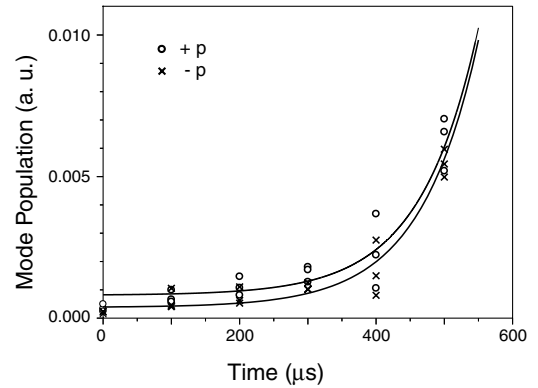


FIG. 3. Amplification of excitations in a BEC with $a < 0$. The growth of the normalized mode population in both $+\mathbf{p}$ and $-\mathbf{p}$ modes as a function of hold time at $a = -1.35$ nm is fitted to an exponential dependence according to Eq. (2), which gave a common growth rate of $5.89 \pm 0.83 \text{ ms}^{-1}$. A variable offset accounted for the initial seeding of the $+\mathbf{p}$ mode.

large error bars reflect the high sensitivity of the dynamics to the magnetic field. In particular, the inelastic loss rates as a function of a have not yet been well characterized, which limits the accuracy of our data.

The results presented here prove conclusively that quantum evaporation is part of the complex dynamics that occur during the collapse of an attractive condensate. While we select a particular mode for observation, the effect is predicted to happen for all modes satisfying $|\mathbf{p}|^2/2m < 2|\mu|$. At short times, the pairwise emission of atoms also implies that the number of atoms in the conjugate mode pairs will be exactly correlated, although as the condensate becomes increasingly depleted, higher order effects will degrade the correlation [5].

Since quantum evaporation is intrinsic to the condensate, a natural question to ask is how large a part it plays in the unperturbed collapse of the condensate. From [5], the depletion of the condensate density n_0 is given by

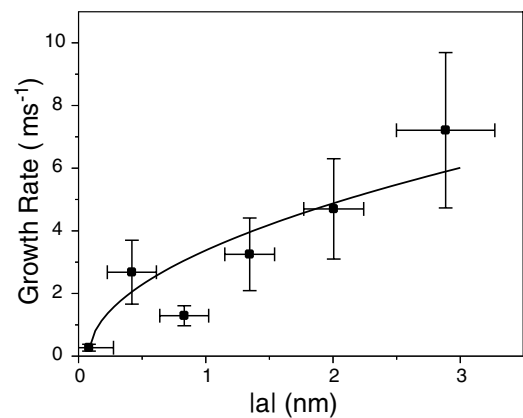


FIG. 4. Growth rate of excitations in a BEC with $a < 0$. The rate increases with the magnitude of the negative scattering length $|a|$. The solid line shows a best fit to the predicted $b_1\sqrt{|a|} - b_2$ dependence.

$$\dot{n}_0 = -4\sqrt{\frac{2\pi\hbar}{mt}}a^2n_0^2\exp(8\pi\hbar|a|n_0t/m). \quad (3)$$

Using our parameters, we extract the time $t_{1/e}$ taken for n_0 to decay to $1/e$ of its original value and compare it to the observed time taken for the condensate to decay without initial seeding until it is no longer discernable (using a visibility threshold of about 10% to 20%) in Fig. 5. As both the number of unstable modes and the amplification rate increase with a , there is a significant decrease of $t_{1/e}$ with $|a|$. We also considered the effect of the initial quantum depletion ($\sim 1\%$, [14]) on $t_{1/e}$, but concluded that its impact is small.

In addition, we compare the observed collapse time to the decay time predicted for global collapse. This implosion corresponds to the unstable monopole mode and may be qualitatively described by Eq. (1) if we assume a momentum equal to \hbar times the inverse size of the condensate. However, the global collapse is more accurately described by accounting for the inhomogeneous spatial profile of the condensate. The inherent pressure gradient will cause the condensate to collapse inwards and the resulting compression causes the density to increase and sharply enhance the three-body recombination loss rate, which goes like n_0^2 . We model the radial evolution using the mean squared radius $R = \int r^2|\psi(r)|^2r dr$. For our cylindrical condensates with an aspect ratio of 100:1, an analytical solution for the resulting 2D dynamics exists [15,16], given by $\dot{R} = 4(\frac{E}{m} - \omega^2R)$, where E is the total energy of the system. Since $\omega = 0$ and $\frac{dE}{dt} = 0$ in our experiments, E is conserved and determined from initial conditions. The time taken to reach $R = 0$ is

$$t_{\text{decay}} = \frac{1}{\omega_0} \sqrt{\frac{a_0}{|a|}}. \quad (4)$$

Equation (4) is also plotted in Fig. 5 and its intersection with $t_{1/e}$ separates the graph into two domains. For small $|a|$, global collapse is predicted to dominate over quantum evaporation while the converse is true at larger $|a|$. This is explained by noting that the decay of the condensate is dominated by the amplification of unstable modes with $\mathbf{p}_{\text{max}} = \sqrt{2m}|c| = \hbar/\xi$, where ξ is the value of the (imaginary) healing length [Eq. (1)]. When ξ is comparable to the condensate size, the local description is invalid and global collapse occurs. In the Thomas-Fermi limit, however, ξ is small and local instabilities deplete the condensate much faster than global collapse. This is consistent with our observations in Fig. 5. We were unable to study the condensate lifetime at even more negative scattering lengths since the condensate then decayed almost instantaneously. To realize a global collapse time much slower than $t_{1/e}$ would require a combination of a weak trap with a high number of atoms, which is currently out of reach.

In conclusion, we have shown that large condensates far in the Thomas-Fermi regime undergo amplification of

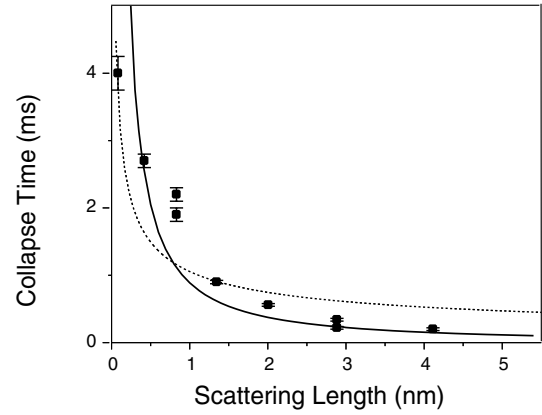


FIG. 5. Time scales for BEC decay. The observed collapse time is compared to the time taken for the condensate density to fall to $1/e$ of its original value ($t_{1/e}$) due to instability amplification [solid curve, Eq. (3)] and the expected t_{decay} for global collapse [dashed curve, Eq. (4)].

local instabilities when their scattering length becomes negative. We have studied the dependence of amplification rate on the magnitude of the negative scattering length and found reasonable agreement with the theory. For our parameters, this quantum evaporation process becomes faster than the global collapse.

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- [1] J. M. Gerton *et al.*, Nature (London) **408**, 692 (2000).
- [2] S. Inouye *et al.*, Nature (London) **392**, 151 (1998).
- [3] E. A. Donley *et al.*, Nature (London) **412**, 295 (2001).
- [4] R. A. Duine and H. T. C. Stoof, Phys. Rev. Lett. **86**, 2204 (2001).
- [5] V. A. Yurovsky, Phys. Rev. A **65**, 033605 (2002).
- [6] E. A. Calzetta and B. L. Hu, arXiv:cond-mat.
- [7] L. Deng *et al.*, Nature (London) **398**, 218 (1999).
- [8] J. M. Vogels *et al.*, Phys. Rev. Lett. **89**, 020401 (2002).
- [9] J. Stenger *et al.*, Phys. Rev. Lett. **82**, 2422 (1999).
- [10] J. Stenger *et al.*, Phys. Rev. Lett. **82**, 4569 (1999).
- [11] We observe the resonance around 1208 ± 10 G, which agrees with the theoretical value within the systematic error of the magnetic field calibration which was done at low fields using rf spectroscopy.
- [12] C. Samuelis *et al.*, Phys. Rev. A **63**, 012710 (2000).
- [13] This is an approximation, since the loss rates for condensate and noncondensate atoms are different due to quantum statistics.
- [14] K. Huang, *Statistical Mechanics* (Wiley, New York, 1987).
- [15] Y. Kagan *et al.*, Phys. Rev. A **54**, R1753 (1996).
- [16] Y. Kagan *et al.*, Phys. Rev. Lett. **79**, 2604 (1997).