

Parametric Amplification of Scattered Atom Pairs

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We have observed parametric generation and amplification of ultracold atom pairs. A ^{87}Rb Bose-Einstein condensate was loaded into a one-dimensional optical lattice with quasimomentum k_0 and spontaneously scattered into two final states with quasimomenta k_1 and k_2 . Furthermore, when a seed of atoms was first created with quasimomentum k_1 we observed parametric amplification of scattered atoms pairs in states k_1 and k_2 when the phase-matching condition was fulfilled. This process is analogous to optical parametric generation and amplification of photons and could be used to efficiently create entangled pairs of atoms. Furthermore, these results explain the dynamic instability of condensates in moving lattices observed in recent experiments.

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Nonlinear atom optics is a novel research area born with the advent of Bose-Einstein condensates of alkali atoms [1]. Unlike photons, ultracold atoms have a very strong nonlinearity directly provided by s -wave collisions, and therefore they do not need a nonlinear medium to provide effective interaction. A number of nonlinear processes first observed with photons have been demonstrated with matter waves such as four-wave mixing [2,3], solitons [4–7], second-harmonic generation [8–11], and sum-frequency generation [9]. Nonlinear atom optics, and, in particular, four-wave mixing, has previously been suggested as an ideal way to create entangled pairs of atoms [3,12,13]. However, in previous four-wave mixing experiments [2,3] using condensates in free space, the quadratic dispersion relation for free particles only allowed for the phase-matching condition to be fulfilled when the magnitudes of all four momenta were equal (in the center-of-mass frame). This is the only way in which two particles can scatter off each other and conserve energy and momentum. In particular, in free space, if a condensate is moving with momentum k_0 , atoms within the condensate cannot elastically scatter into different momentum states, and therefore the analog to optical parametric generation of photons is not possible.

The situation is very different when an optical lattice is added. The lattice delivers energy in the form of the ac Stark effect and momentum in units of $2\hbar k_L$ to the atoms, where k_L is the wave vector of the optical lattice. The motion of atoms in this periodic potential is described by a band structure, which deviates from the quadratic free particle dispersion curve. In a lattice, as recently suggested [14], it becomes possible for two atoms in the condensate to collide and scatter into a higher and lower quasimomentum state and conserve energy. As we discuss below, this can lead to dynamic instabilities of the condensate, but also enables nondegenerate four-wave mixing and the atom-optics analog of optical parametric generation.

Phase matching is essential for high efficiency in nonlinear processes in quantum optics including optical para-

metric generation of photons [15], and a modification of the dispersion curve has been used to demonstrate optical parametric amplification in semiconductor microcavities [16]. In atom optics, dispersion management was used to modify the effective mass of atoms [17], and to create bright gap solitons [18]. Here we demonstrate that by modifying the dispersion curve using an optical lattice, scattering processes which cannot occur in free space become possible, and we realize the matter-wave analogue of an optical parametric generator (OPG) and an optical parametric amplifier (OPA).

To demonstrate the matter-wave analogue of an OPG, a ^{87}Rb Bose-Einstein condensate with quasimomentum k_0 was loaded into a one-dimensional optical lattice. To load the atoms at a given quasimomentum relative to the Brillouin zone, a moving optical lattice was adiabatically applied to a magnetically trapped condensate initially at rest in the lab frame. The lattice was created using two counter-propagating laser beams with frequency difference $\delta\nu$, giving the lattice a velocity of $v = \frac{\lambda}{2} \delta\nu$, where λ is the wavelength of the optical lattice. In the rest frame of the lattice, the condensate has quasimomentum $k_0 = \frac{m\lambda}{2\hbar} \delta\nu$, where m is the atomic mass. By changing the detuning between the lattice beams, $\delta\nu$, k_0 could be varied. As shown in Fig. 1(d), as the value of k_0 was varied we observed elastic scattering of atom pairs into final states k_1 and k_2 . The range of possible final states varied with k_0 due to the phase-matching condition. For values of k_0 less than $\approx 0.55k_L$ the dispersion relation imposed by the Bloch structure of the optical lattice does not allow elastic scattering to occur. For our lattice depth of $V = 0.5E_{\text{rec}}$, where $E_{\text{rec}} = \hbar^2 k_L^2 / 2m$, the values of k_2 which satisfied energy and momentum conservation were beyond the first Brillouin zone. Since the scattering process occurs within the first Bloch band, the atoms in state k_2 have a quasimomentum $k_2 = (2k_0 - k_1) \text{Mod}(2k_L)$ [see Fig. 1(a)]. As the value for $\delta\nu$ (and the resulting value of k_0) is increased, the separation between k_0 and the allowed states k_1 and k_2 decreases as is clearly observed in Fig. 1(d). For values of

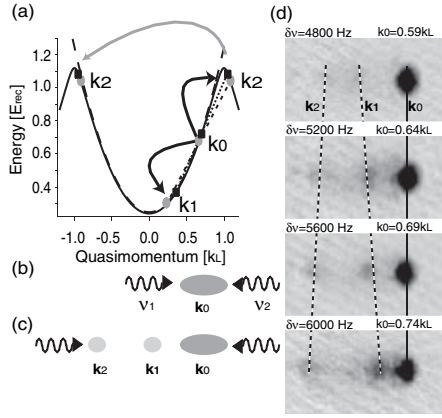


FIG. 1. Dispersion curve for the optical lattice and experimental setup. (a) Band structure for a lattice depth of $V = 0.5E_{\text{rec}}$. The dashed line shows the free particle dispersion curve. The dispersion relation of the lattice allows two atoms with momentum k_0 to elastically scatter into the final momentum states k_1 and k_2 . Energy and quasimomentum are conserved when k_0 is the average of k_1 and k_2 and the three points on the dispersion curve lie on a straight line. If k_0 is varied, the allowed values for k_1 and k_2 change. For values of k_0 below $\approx 0.55k_L$, where k_L is the wave vector of the optical lattice, atoms cannot scatter elastically into different momentum states. The circles (squares) show allowed states k_0, k_1, k_2 for $k_0 = 0.66k_L$ ($0.70k_L$). As k_0 is increased, the final momentum states move closer together. Since the scattering occurs within the lowest band of the lattice, the final momentum is $k_2 = (2k_0 - k_1) \text{Mod}(2k_L)$. (b) A ^{87}Rb Bose-Einstein condensate is illuminated by two counter-propagating laser beams with detuning $\delta\nu$, which create a moving optical lattice. The condensate is initially held at rest. In the rest frame of the lattice, the condensate has quasimomentum $k_0 = \frac{m\lambda}{2h} \delta\nu$. (c) As k_0 was varied, we observed elastic scattering into states k_1 and k_2 . (d) Absorption images for different lattice detunings, $\delta\nu$, showing parametric generation. After ramping up the lattice, the atoms were held for 10 ms at a constant lattice depth. They were then released from the trap and imaged after 43 ms of ballistic expansion. The field of view is $0.5 \text{ mm} \times 0.3 \text{ mm}$.

k_0 above $\approx 0.75k_L$, the final momentum states were no longer distinguishable, and the condensate became unstable.

To demonstrate the matter-wave analogue of an OPA, we first created a small seed of atoms with quasimomentum k_1 before ramping on the moving lattice (see Fig. 2). To create the seed we applied a Bragg pulse to the magnetically trapped condensate, outcoupling a small fraction of atoms into the momentum state k_{Bragg} [19]. Immediately after applying the pulse, the optical lattice was adiabatically ramped on. In the rest frame of the lattice, the seed has quasimomentum $k_1 = k_{\text{Bragg}} + k_0$. The phase-matching condition for a given seed k_{Bragg} was found by varying the frequency difference $\delta\nu$ of the lattice, and therefore the quasimomenta k_0 and k_1 of the atoms. As shown in Fig. 2(d), when the phase-matching condition was fulfilled, we observed amplification of the seed k_1 as

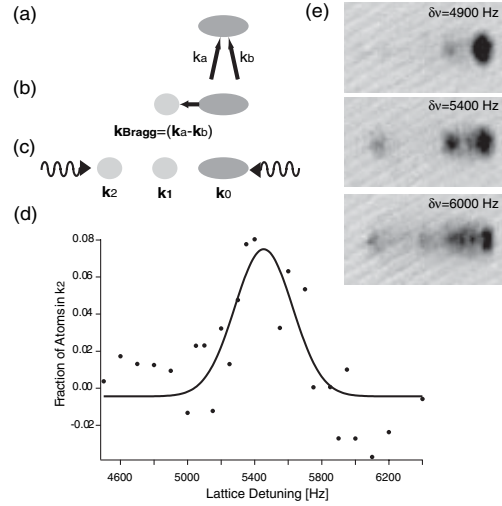


FIG. 2. Parametric amplification of scattered atom pairs in a 1D optical lattice. (a) First, a 2 ms Bragg pulse was applied to the condensate. (b) The Bragg pulse seeded atoms along the long axis of the condensate with momentum $k_{\text{Bragg}} = (k_a - k_b)$ in the lab frame. (c) The optical lattice was then adiabatically ramped on and applied for 10 ms. When the phase-matching condition was fulfilled, parametric amplification of atoms in the seeded state k_1 and its conjugate momentum state k_2 was observed. (d) Resonance curve showing amplification of k_2 , when k_1 was seeded. Amplification occurred only when the phase-matching condition was met. For a fixed k_{Bragg} , the resonance condition was found by varying the detuning $\delta\nu$ of the lattice. The data was taken for $k_{\text{Bragg}} = 0.43k_L$. The fraction of amplified atoms was obtained by subtracting images with and without the seed pulse. (e) Absorption images showing amplification of k_1 and k_2 when the phase-matching condition is met. The center of the resonance was at $\delta\nu \approx 5450$ Hz, close to the calculated value of $\delta\nu \approx 5350$ Hz. The width of the resonance is determined by the Fourier width of the Bragg pulse. Most of the scattered atoms in the third image were independent of the seed pulse.

well as its conjugate momentum k_2 . The growth of k_1 and k_2 as a function of time are shown in Fig. 3.

The experiments were performed using an elongated ^{87}Rb condensate created in a magnetic trap previously described in Ref. [20]. The magnetic trap had a radial (axial) trap frequency of 35(8) Hz. The condensate, containing between $0.5\text{--}3.0 \times 10^5$ atoms, was produced in the $|5^2S_{1/2}, F = 1, m_F = -1\rangle$ state. The Bragg pulse was created with two laser beams derived from the same laser, which was red detuned from the $5^2S_{1/2}, F = 1 \rightarrow 5^2P_{3/2}, F = 1$ transition at $\lambda = 780 \text{ nm}$ by 400 MHz, and was π polarized. As shown in Fig. 2, the Bragg beams were aligned such that atoms were outcoupled along the long axis of the condensate. The intensity of the Bragg pulse was chosen such that less than 5% of the initial condensate was outcoupled into k_{Bragg} , and the length of the pulse was 2 ms. The angle between the Bragg beams could be varied to change the momentum of the outcoupled atoms. The optical lattice was created using two counter-

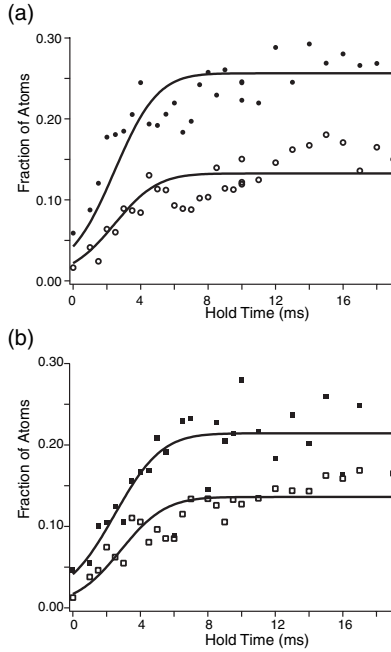


FIG. 3. Growth curve for atomic population in quasimomentum states k_1 and k_2 when the process was seeded. (a) Amplification of atoms with quasimomentum k_1 (solid points), and with the conjugate momentum k_2 (open points), when state k_1 was seeded. (b) Amplification of atoms in k_1 (solid points), and k_2 (open points), when k_2 was seeded. The values for k_0, k_1, k_2 were $0.66k_L, 0.23k_L,$ and $-0.92k_L$ respectively. The solid lines shows the expected gain using Eq. (1) with variable scale factors for each curve as the only free parameters.

propagating beams derived from the same laser with $\lambda = 1064$ nm, and the frequency of the two beams were controlled by two separate acousto-optic modulators (AOMs) driven with a frequency difference $\delta\nu$. The lattice was also aligned along the long axis of the condensate, and was ramped on in 1 ms using an exponential ramp. After the condensate was held in the lattice for a variable time τ it was then released from the trap and imaged after 43 ms of ballistic expansion.

For all of our experiments, the depth of the optical lattice was $V = 0.5E_{\text{rec}}$ with a band structure shown in Fig. 1. When the process was not seeded, atoms were elastically scattered into a narrow band of states k_1 and k_2 , where both energy and momentum were conserved. However, the population in neither state was large enough for amplification to be observed. When the process was seeded, amplification occurred when the quasimomentum was tuned such that energy and momentum were conserved for the states $k_0, k_1,$ and the conjugate momentum k_2 . In our experiment, the difference $\Delta k = k_0 - k_1$ between the quasimomenta of the condensate k_0 and seed k_1 was set by the angle of the initial Bragg pulse. For a given Bragg angle, there is only one set of quasimomenta $k_0, k_1,$ and k_2 where the phase-matching condition is fulfilled. To find this point, we varied the velocity of the moving lattice for fixed hold

times. Results for $k_{\text{Bragg}} = 0.43k_L$ are shown in Fig. 2. The phase-matched value for k_2 is at $1.08k_L$, beyond the boundary of the first Brillouin zone. Therefore, the atoms are observed with a momentum $k_2 = -0.92k_L$. For $k_{\text{Bragg}} = 0.43k_L, 0.34k_L,$ and $0.28k_L$, we observed resonances at $\delta\nu = 5450$ Hz, 5750 Hz, and 6100 Hz, respectively. For these Bragg angles and our lattice depth, the expected values were 5350 Hz, 5700 Hz, and 6050 Hz.

In Fig. 3, 5% of the initial condensate containing $N_0 = 1.3(3) \times 10^5$ atoms was outcoupled with $k_{\text{Bragg}} = 0.43k_L$. The gain for the process is determined by the strength of the nonlinear interaction $U = \frac{4\pi\hbar^2 a}{m}$ between atoms in the condensate, where a is the s -wave scattering length. We can estimate the maximum amplification rate to be $\eta = 2n_0U/\hbar$ [3], with $\dot{N}_{1(2)} = \eta N_{2(1)}$, where $N_{1(2)}$ is the number of atoms in the momentum state $k_{1(2)}$, and n_0 is the condensate density. For $N_0 = 1.3(3) \times 10^5$, the maximum growth rate should be $\eta = 540$ Hz. The amplification rate will decrease as the state k_0 is depleted. However, for our small seeds, the amplification was limited by the loss of overlap between the condensate and the amplified pulses. The Thomas-Fermi radius (R_{TF}) of the condensate in the axial direction was $33 \mu\text{m}$, and the recoil velocity (v_{rec}) for the final states k_1 and k_2 with respect to the initial condensate was $v_{\text{rec}} = 1.8 \mu\text{m/ms}$ and $6.8 \mu\text{m/ms}$, respectively. The overlap integral between the amplified atoms and the initial condensate can be approximated as a Gaussian with time constant $\tau_c \approx 0.75R_{\text{TF}}/v_{\text{rec}}$, which for our parameters is 3.75 ms. We compare our results to the modified rate equation

$$\dot{N}_{2(1)} = \eta N_{1(2)} e^{-t/\tau_c}. \quad (1)$$

Since atoms are scattered into states k_1 and k_2 in pairs, one would expect that the final atom number in the two states (minus the initial seed) are equal. Instead, we observe a smaller number in state k_2 which we ascribe to the proximity of k_2 to the boundary of Brillouin zone. This leads to instabilities, where atoms in state k_2 are scattered into other momentum states or into higher bands. If we allow a variable scale factor in our model to correct for the loss of atoms in N_2 , as shown in Fig. 3, the gain for $N_{1,2}$ is in agreement with the experimental data.

Amplification was also observed when atoms were seeded in state k_2 . Because of the geometry of our experimental setup, we were unable to load atoms directly into $k_2 = -0.92k_L$. However when atoms with quasimomentum $k = 1.08k_L$ were loaded into the lattice, the ramp-up was no longer adiabatic due to their proximity to the boundary of the first Brillouin zone. Because of this, atoms from the seed were loaded into both the second Bloch band (with $k = 1.08k_L$) and the ground state (with $k = -0.92k_L = k_2$). As shown in Fig. 3(b), the gain for this process was almost identical to when atoms were seeded in state k_1 .

The loss of overlap could be alleviated by using a more extreme trap geometry in which the condensate is more elongated, e.g., by confining atoms in a tight transverse optical lattice. In this configuration, it may be possible to observe the parametric scattering dynamics for longer time scales, which may allow for the observation of Rabi oscillations between k_0 and k_1, k_2 as predicted in Ref. [14]. For longer coherence times, parametric amplification could also be an efficient means of producing pairs of momentum entangled atoms for quantum information applications [12,13], but the issue of secondary collisions out of states k_1 and k_2 has to be carefully addressed.

For high atom numbers, and for large values of k_0 , the condensate became unstable and scattered into a broad band of final momentum states [Fig. 2(e)]. For $k_{\text{Bragg}} = 0.43k_L$, the energy of atoms outcoupled by the Bragg beams was ≈ 370 Hz, whereas the chemical potential of the condensate was ≈ 300 Hz. Because of this, if the atom number was increased significantly the momentum peaks were no longer distinguishable. When the chemical potential of the condensate was larger than the separation between the phase-matched momentum states, the process was self-seeded; i.e., the momentum spread of the initial condensate contained atoms with momentum k_0, k_1 , and k_2 , and considerable scattering occurred. Similarly, if the atom number was kept constant, and the value of k_0 was increased, the phase-matched momentum states moved closer together until they were no longer distinguishable. This occurred at values of k_0 above $\approx 0.75k_L$, and we observed a dynamic instability. For larger atom numbers, the critical value of k_0 decreases. For values of k_0 less than $0.55k_0$ elastic scattering cannot occur, and the system should be stable for all atom numbers. Instabilities of condensates in optical lattices has attracted much attention recently, both theoretically [21–28] and experimentally [29–33]. Most recently, dynamic instabilities of condensates in moving lattices were observed in Refs. [29,30]. In Ref. [30], the chemical potential was a factor of 3 higher than in our experiment, leading to a dynamic instability for all values of k_0 above $0.55k_L$. Although discrete momentum states could not be observed in those experiments, it is possible that the mechanism for the dynamic instability is self-seeded parametric amplification. Indeed, the phase-matching condition for parametric amplification is identical to the resonance condition for dynamic instability in the noninteracting limit [21]. After the submission of this Letter, recent work on period-doubling instabilities in a shaken optical lattice [34] was reinterpreted as parametric amplification [35].

In conclusion, we have demonstrated a matter-wave analogue of both optical parametric generation and optical parametric amplification using a condensate moving in a one-dimensional optical lattice. The optical lattice modified the dispersion curve and ensured phase matching. If the separation of the phase-matched momentum states

becomes less than the speed of sound, a condensate will self-seed the process and become dynamically unstable.

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