Auxiliary Material: Determination of the Superfluid Gap in Atomic Fermi Gases by Quasiparticle Spectroscopy

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Determination of the superfluid boundary

It has been shown previously [1] that at unitarity the difference column density profiles serve as an indicator for the SF-N boundary. The discontinuity of the minority density results in a pronounced "cusp" in the difference profile $n_\uparrow(r) - n_\downarrow(r)$, see fig. 1. In the main body of the paper the phase boundary has been determined by this peak position. Hence, "normal" refers to spatial regions beyond the peak, "superfluid" refers to spatial regions inside.

The color coding in the graphs in fig. 1 shows where the spectral overlap (definition see below) between the majority and minority pairing peaks is lost. On resonance ($B = 690$ G) this position shows excellent agreement with the position of the cusp in the difference density profile and can therefore serve as an alternative indicator for the SF-N boundary. This coincidence breaks down away from resonance: On the BCS side ($B = 710$ G) the spectra are less "robust" against polarization and spectral overlap is lost before the column density difference shows a peak. The reverse situation occurs on the BEC side of the resonance ($B = 671$ G). Note that on the BEC side the minority cloud does not extend much further than the peak position in the column density difference.

Quantification of spectral overlap

In order to calculate the spectral overlap of a spectrum, the quasiparticle peak was fitted by a gaussian and subtracted from the majority spectrum. The overlap is then defined as one minus the difference of the integrated spectra normalized by the sum of the integrated spectra, see fig. 2. As mentioned in the main text and above, on the BEC side of the Feshbach resonance almost complete spectral overlap can be observed into the normal region. On the BCS side the reverse situation occurs.

Minority peak shift for high imbalance

Fig. 4 in the main body of the text shows the peak positions normalized by the local majority Fermi energy $\epsilon_{F\uparrow}$. Fig. 3 shows the bare peak positions as observed in the experiment.

One unexpected finding in fig. 4a in the main body of the text is the sudden increase of the minority peak position for high imbalances. This behavior can be traced back to the data in fig. 3: The minority peak position (red) shows a change in curvature as the SF-N boundary is crossed, in contrast to the majority Fermi energy. Therefore, the ratio of minority peak and majority Fermi energy shows a sudden increase towards the edges of the cloud. The value of the peak position of $\omega_{pol} \simeq 0.9\,\epsilon_{F\uparrow}$ (where final state interactions of $E_{final} \simeq 0.05\,\epsilon_{F\uparrow}$ have been taken into account) is higher than the theoretically calculated value of $\simeq 0.6\,\epsilon_{F\uparrow}$ [2].

Quasiparticles in an equal mixture

In a previous publication [3] low temperature RF spectra did not show any signatures of quasiparticles. We have attempted to create thermally generated quasiparticles in an equal density mixture for higher temperatures $T/T_F \geq 0.20$. The experimental results in fig. 4 show a decrease of the gap parameter, but no local double peak structures could be resolved for any temperature.

We can estimate the temperature required to populate quasiparticles, assuming that the temperature has to be on the order of the gap and that $\Delta(T)$ is given by the BCS relation $\frac{\Delta(T)}{\Delta_0} = 1.74\sqrt{1 - \frac{T}{T_c}}$ [4]. At unitarity, $\Delta_0/T_c \simeq 3$. One would expect to populate quasiparticles only very close to the transition temperature $T/T_c \simeq 0.95$, when the gap is only one third of its low-temperature value.

The theoretical dissociation spectrum including the Hartree term U

Starting from a BCS-Leggett mean-field wavefunction and applying Fermi's Golden Rule, the RF spectrum can be described by (neglecting the Hartree term):

$$\Gamma(\omega) \propto \frac{\sqrt{\omega - \omega_{th}}}{\omega^2}\sqrt{1 + \frac{\omega_{th}}{\omega} + \frac{2\,\mu}{\omega}} \tag{1}$$

where $\omega_{th} = \sqrt{\Delta^2 + \mu^2} - \mu$ is the dissociation threshold and μ is the chemical potential.

The corresponding quasiparticle dispersion relation is:

$$E_k = \sqrt{\Delta^2 + (\epsilon_k - \mu)^2} \tag{2}$$

ϵ_k being the free particle dispersion $\epsilon_k = \frac{\hbar^2 k^2}{2m}$.

Hartree terms U modify the quasiparticle excitation spectrum [5, 6]:

$$E_k = \sqrt{\Delta^2 + (\epsilon_k - (\mu - U))^2} \tag{3}$$

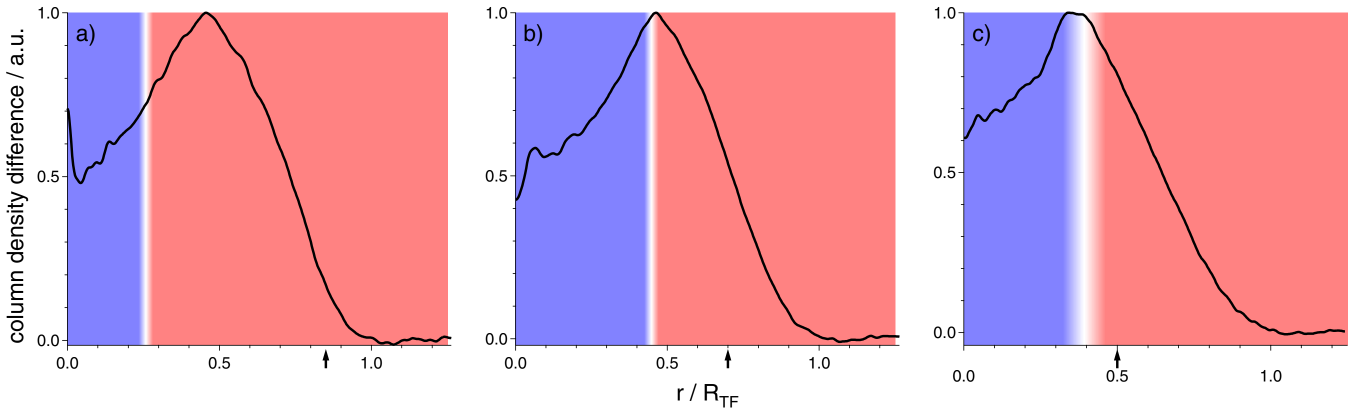

FIG. 1: (Color online) The difference column density profile $n_\uparrow(r) - n_\downarrow(r)$ (radially averaged). a) BCS side ($B = 710G$), b) Unitary limit ($B = 690G$), c) BEC side ($B = 671G$). Each profile shown is the average of 10 individual profiles. Blue marks the region of complete spectral overlap between majority and minority components, red marks the region where there is no complete spectral overlap. The black arrows at the bottom indicate the radial size of the minority component.

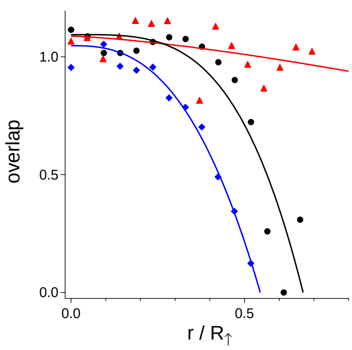

FIG. 2: (color online) Overlap of majority and minority pairing peak as a function of position in the trap for various interaction strengths. A power law was fitted to the curves as a guide to the eye. Unitary limit (black circles): $1/k_{F\uparrow}a = 0$, phase boundary at $r_c/R_\uparrow \simeq 0.46$; BEC side (red triangles): $1/k_{F\uparrow}a = 0.39(1)$, $r_c/R_\uparrow \simeq 0.45$; and BCS side (blue diamonds): $1/k_{F\uparrow}a = -0.25(1)$, $r_c/R_\uparrow \simeq 0.35$

resulting in an RF spectrum of

$$\Gamma\left(\omega'\right) \propto \frac{\sqrt{\omega' - \omega'_{th}}}{\omega'^2} \sqrt{1 + \frac{\omega'_{th}}{\omega'} + \frac{2\,\mu'}{\omega'}} \qquad (4)$$

where $\omega' = \omega + U$, $\omega'_{th} = \omega_{th} + U$ and $\mu' = \mu - U$. This demonstrates that the spectrum retains its functional form but the entire spectrum is shifted by U.

Resolution / Experimental broadening

For comparison with the theoretical spectrum, we model the RF pulse of $T = 200\ \mu$s length as a square pulse, in the frequency domain, resulting in a FWHM of the RF spectral power of $\Delta\nu = 2 \cdot \frac{1}{2\pi}\, 1.39\, \frac{2}{T} \simeq 4.4$ kHz.

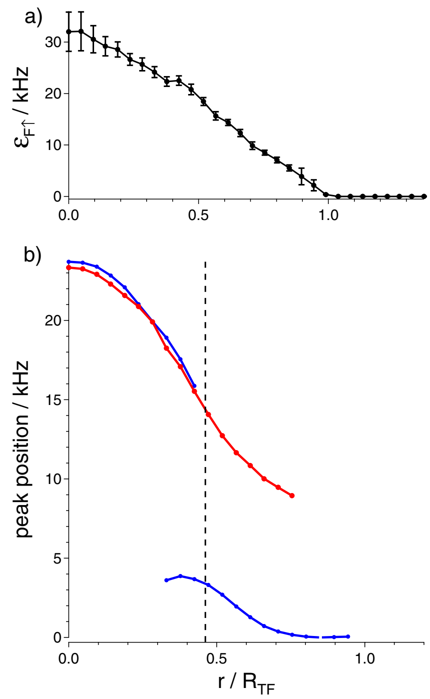

FIG. 3: (color online) a) Local majority Fermi energy in kHZ, the error bars are the standard deviation of the mean value. b) Peak positions of majority and minority in kHz. Majority (Blue): Pairing peak (higher frequencies, only discernible in the SF region) and quasiparticle peak (lower frequencies). Minority (red): The peak can be traced well into the normal region.

The theoretical spectrum consists of two parts: 1) A dissociation peak including the Hartree energy, described by eqn. 1 with Δ and U as given in table I in the main text. 2) A quasiparticle peak modeled as a narrow (FWHM = 1 kHz) Lorentzian with a peak height adjusted so that

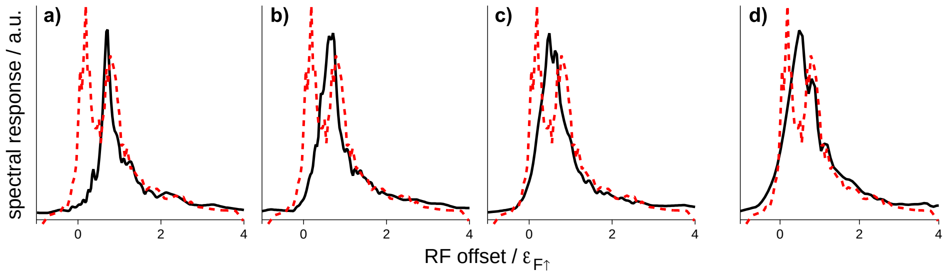

FIG. 4: (Color online) Local RF spectra of an equal spin mixture for various normalized local temperatures $T/T_{F\uparrow}$. a) $T/T_{F\uparrow} \simeq 0.20$, b) $T/T_{F\uparrow} \simeq 0.22$, c) $T/T_{F\uparrow} \simeq 0.34$, d) $T/T_{F\uparrow} \simeq 0.55$. No local double peak spectrum can be resolved in the RF spectrum. For comparison, the double peak spectrum of an imbalanced mixture with $T/T_{F\uparrow} \simeq 0.06$ is added with a red dashed line.

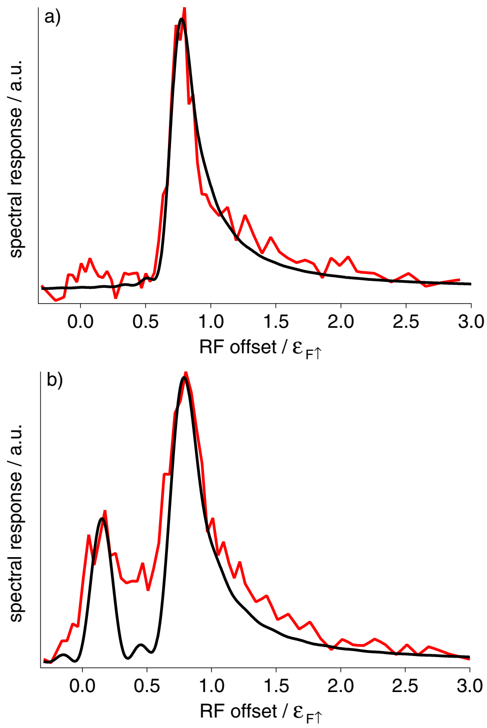

FIG. 5: (color online) Comparison of experimental (red) and theoretical (black) line shapes for spectra a) and b) of figure 1 of the main body of the paper. The theoretical curve is obtained from a BCS-Leggett mean field description including the Hartree term and a convolution with the experimental resolution of $\simeq 4.4$ kHz. The values for Δ and U as calculated from the peak positions lead to a reasonable agreement with the data.

it resembles our data. This spectrum was convolved with the Fourier transform of a square pulse $f(\omega) \propto \frac{\sin^2 \frac{\omega T}{2}}{\left(\frac{\omega T}{2}\right)^2}$. Fig. 5 shows that the theoretical spectrum reproduces our data quite well. The deviation in 5b) might be attributed to additional broadening mechanisms like finite quasiparticle lifetime, finite temperature and atomic diffusion during the duration of the RF pulse. The convolution causes a small shift of $0.05\,\epsilon_{F\uparrow}$ in the spectral peak position due to the asymmetry of the theoretical spectrum and has been accounted for in the determination of Δ and U.

The value given above for the experimental resolution is confirmed by looking at the blurring of the sharp onset of pair dissociation. Equation 1 predicts that the threshold and peak position in the strongly interacting regime differ by less than 10% of the Fermi energy. Adjusting the experimental resolution to ~ 4 kHz accounts for the experimentally observed difference of threshold and peak position of about $0.3\,\epsilon_{F\uparrow}$.

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